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The nuclear explosive yields at Hiroshima and Nagasaki

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THE NUCLEAR EXPLOSIVE YIELDS AT HIROSHIMA AND NAGASAKI

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CONTENTS

The nuclear explosive yields at Hiroshima and Nagasaki have been calculated from measurements of the damage caused to some objects whose dynamical responses were simple enough to permit analysis. Examples include bent or snapped poles, squashed empty drums and cans, overturned memorial stones, some safe doors and the tops of office cabinets pushed in by the blast. The Hiroshima explosion was 12 ± 1 kilotons and the Nagasaki explosion was 22 ± 2 kilotons.

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1. Introduction

When President Truman announced the attack on Hiroshima with an atomic bomb he stated that 'the bomb had more power than 20 000 tons of TNT'. The attack on Nagasaki followed a few days later. At the time when the President made his announcement, only one atomic explosive device had been tested. The design of the bomb which was exploded at Nagasaki was similar to, but not identical with, that of the atomic device which had been tested. Scientific measurements made at the test showed that the energy released (yield) was about the same as the detonation energy of 20 000 tons of TNT (20 kT)†. As a matter of present definition, a yield of 1 kiloton is the rapid release of 10^{12} calories of energy from the fission process into the near environment (equivalent to the fission of 1.45×10^{22} nuclei at 180 MeV per fission). This omits the kinetic energy of neutrinos and the fission-product heat generated later than a few milliseconds after fission. The detonation energy of TNT is about 10^3 calories per gramme or 10^{12} calories per kiloton.

The design and materials of the weapon used at Hiroshima were quite different from those of the weapon used at Nagasaki but the explosive effects in the two cases were expected to be roughly the same. On the authority of the President's statement about Hiroshima, the yield of the nuclear weapon used at Hiroshima has always been quoted as 20 kT; and it has been widely assumed that Nagasaki was about the same. There has never seemed to be any need to attempt to set limits of accuracy on the two yields. Even if a case had been thought to exist for reasons of historical accuracy, there were no more accurate values available, nor was it obvious that it was possible to make better estimates.

The biological consequences which nuclear weapons would cause were not fully understood at the time when the weapons were used. Less was known about the genetic and possible other long-term effects than about the more immediate effects of radiation sickness. A large Japanese–United States effort has been applied to treating, studying and collating the biological effects caused in the two cities. Work by Japanese biologists and doctors began within a few hours of the Hiroshima explosion and the joint effort by Japan and the United States started a few weeks later; and has continued ever since. There is still much work to be done.

Obviously, the interpretation of these biological data and their collation with other biological data require a fairly accurate knowledge of the γ-radiation and neutron doses, and their spectra, throughout the two cities. Biological effects with the same dose show variations from individual to individual, and a small percentage uncertainty in the magnitude of the dose, or its exact make-up, would not be important. A greater uncertainty would cause the biological correlations to lose some of their significance.

The scientific measurements made at the first test of a nuclear device and of later tests are given in the book prepared and published by the United States Atomic Energy Commission. *The effects of nuclear weapons* (U.S.A.E.C., first, second and third editions). Since the Nagasaki weapon was nearly the same in its explosive components as the first device that was tested, the data in the book can be used as the basis for estimating doses at any particular position in Nagasaki at the time of the explosion, correcting factors being applied for the effects of walls and buildings, and for floor level and other parameters, such as atmospheric density.

The estimation of doses in Hiroshima needs additional information. Whatever value is

† For historical and practical reasons (see p. 360) several non-metric units are used in this paper instead of the SI units now normal in the Society's journals.

considered correct for the yield, the 'open-site' γ-radiation and neutron doses at different points in the city will not be immediately obtainable from the data given in the U.S.A.E.C. book. It will still be necessary to make allowances for the particular design of the Hiroshima weapon.

The many difficult problems of interpreting and correlating the biological data would clearly be helped considerably by an estimate of the Hiroshima yield, accurate to within say $\pm 10\%$. A corresponding estimate of the Nagasaki yield, establishing that the yield was about 20 kT, would confirm the expectation that the yield was about the same as that of the similar test explosion; and since there is not much doubt that the yield was about the same, would strengthen confidence in the method of estimation. The only practical possibility today which might enable such estimates to be made is to revise in the light of modern information any observations made on the blast waves.

A team from the Manhattan District was sent to Tokyo the day after the entry of General McArthur into that city, and soon thereafter spent a few days both at Nagasaki and Hiroshima. The team was divided into small groups, and each was given a task. One of the authors of the present paper (W. G. P.), together with two American colleagues (Professor R. Serber and Ensign G. T. Reynolds, U.S.N.) were asked to report on the physical damage. It was impossible for three people in a few days to make more than the most cursory examination of the damage to buildings; and it was obvious that large specialized teams would soon be coming to make detailed studies, item by item, and building by building. After a day or two of almost pointless work, a new idea emerged. This was to look systematically for any simple damaged object which would enable calculations to be made about the parameters of the blast wave. At first, it was thought that the most promising method would be to draw contours of damage effects to buildings and other types of structure and then compare the two explosions. This method was quickly found to be valueless because of the difficulty of finding a number of comparable structures at various distances from ground zero and setting a damage scale. The average snapping distance of telegraph poles was too dependent on the dimensions of the pole and on the cross-piece and wires. Flash burns to telegraph poles seemed more definite but there were variations.

The observers' attention gradually became focused on bent cylindrical flagpoles and other drag damage effects; dished or broken panels, one side exposed to the air pressure from the blast and the other side fully or almost protected; partially collapsed empty cans with little or no openings; and any other damage effects which might permit estimates of the blast to be made. Measurements were made on the spot and a variety of samples were collected. Specimens were taken back to England by W. G. P. for tests, and a report was written about the physical effects of the two nuclear explosions on buildings and materials and on the estimated magnitudes of the explosions.

The report was required quickly and time did not permit more than a superficial analysis of some of the dynamic responses of the objects which had been studied to determine the magnitude of the blast which had affected them. The maps used for obtaining distances were not accurate. The points chosen for the two ground zeros were some 100 ft from the points determined by later teams who made very careful measurements. Because of these various deficiencies, and because the basic blast data and some of the drag coefficients at the time involved large errors, the conclusions drawn about the magnitudes of the explosions could not now be considered reliable. The conclusions reached were that (in today's terminology) the yield of the Hiroshima

explosion was approximately 10 kT, and the yield of the Nagasaki explosion was greater, and might have been three to four times larger.

The present paper is mainly concerned with explaining how these observations have recently been re-examined by using more accurate data in order to make the best possible estimates of the yields of the two explosions. In one or two minor respects the observations have been supplemented by observations described in other reports on the damage in the two cities; but as far as we know there are few facts recorded susceptible to an analysis which determines accurately the magnitudes of the blast waves or the yields of the explosions.

The best choice of units to be used in this paper was not obvious. The U.S.A.E.C. book, *The effects of nuclear weapons*, gives formulae relating to blast waves and many tables and graphs of basic blast data. The formulae are useful in our work and the basic data are essential, although some small but significant corrections are needed. We decided to use the same notation and units as those in the book, and these are the same as those used in the original observations.

The interpretation of the yields of the two explosions would be falsified if precursors had formed. Since the height of burst of the Nagasaki explosion scaled to 1 kT is appreciably less than the height of the Hiroshima explosion scaled to 1 kT, a precursor was more likely at Nagasaki than at Hiroshima, but the information given in the 1964 edition of the U.S.A.E.C. book establishes that precursors were not formed.

It should be mentioned that in the original 1945 report, an attempt was also made to relate the yields of the two explosions by the relative distances at which scorching of telegraph poles could be detected. The results were too variable to give any confidence of precision. The three Manhattan District observers thought that they had located the critical distance when poles slightly further away showed no signs of scorching; but then other poles still further away appeared to show signs of scorching. The observers considered that the critical distance was 9500 ft in Hiroshima and 11 000 ft in Nagasaki. The U.S.A.E.C. book gives 9000 ft and 11 000 ft respectively. (The two sets of data may not be independent.) To make a decision whether or not there were signs of scorching involved a subjective judgement which was extremely difficult to make. Moreover, the colour and the moisture content of the surface layer would have had a marked influence. We therefore do not accept yield estimates made from the flash burns on telegraph poles. If one now followed the U.S.A.E.C. book and assumed that visible signs of scorching would be given by 5 cal/cm^2, the Hiroshima yield would be 25 kT and Nagasaki nearly 40 kT. These values are certainly too high.

The nuclear explosive yields would not be quite in the inverse ratio of the square of the distances: the longer duration of the larger explosion is also involved in a complicated way.

In this paper we shall not attempt to interpret all of the damage observations made by the three Manhattan District observers. Reinforced concrete floors, with a basement room below, were dished in the Administration Building of the Torpedo Factory at Nagasaki and in the Radio Building at Hiroshima. Details were taken of the reinforcements of the floor panels, and of the amount of dishing, but we have not convinced ourselves that it is possible to make a good estimate of the blast parameters from these observations. Certainly, the estimates we have made are roughly consistent with estimates made from other damage observations, but the margin of uncertainty is so wide that we consider the results are not worth presenting.

There was no consistency in the long-range damage, but in the few cases where the peak overpressure was estimated from a particular observation, the magnitude was considerably less than the value read off theoretical curves. Thus, some paper panels in a bamboo frame failed at

12 000 ft in Hiroshima. The screen was 'side-on' to the blast. The owner of the house explained that he was sitting indoors and saw the whole screen blow in, with many of the paper panels bursting. About 80 % of the paper panels had burst. The rest were intact and samples were taken to England for test. The deduced maximum overpressure was only 0.48 lbf/in.^{2} (1 pound (force) per square inch = 6895 N m^{-2}.)

Similarly, a low peak overpressure of 0.28 lbf/in^2 at a distance of 15 000 ft was deduced from the failure of one panel of the end wall of a timber barn which was effectively air tight against the blast.

A reflexion or refraction phenomenon was described to the three observers by some Japanese at Mogi, about 7 miles from the Hiroshima explosion. Witnesses said that they experienced a big flash, then a loud roar, followed at several second intervals by half a dozen loud reports, from all directions. These were probably reflexions from the hills surrounding Mogi, rather than refracted or reflected waves from different layers in the atmosphere.

2. Ground zeros and the heights of burst

The three Manhattan District observers had no difficulty in deciding the positions of the ground zeros within about 100 ft. However, only by making more precise measurements and using statistical methods has it been possible for later observers to determine the ground zeros with greater precision. The U.S.A.E.C. has provided us with the best data now available, together with good maps of the two cities as they were at the times of the explosions.

There were many flash burns and flash shadows which pointed to where the bursts occurred. The edges of the burns and shadows usually did not have the precision of a ruled line, and usually the edge was not long enough to measure angles better than within a few degrees. One example was found in Hiroshima where exceptional accuracy was possible.

The walls of the Post Office Building about 5100 ft south of ground zero in Hiroshima were covered with a dull white beaver board matted material and the flash of the explosion coming through the windows singed large areas of the walls. Clear unsinged shadows of the steel window frames and of the blind cords were left on the walls. Figure 1, plate 2, is a photograph of some of these shadows. The centre line of the shadows could be determined with good precision and the shadows were a few feet long.

Two of the shadows were measured as accurately as the definition of the edges allowed. On a vertical of 18.0 in the horizontal distance was 43 in: on a vertical of 15.0 in the horizontal distance was 37 in. If the shadow on the wall makes an angle θ with the horizontal, then these two measurement give θ as 22° 43′ and 22° 4′, with an average of 22° 23′.

The shadows were on the third floor above the ground floor and were therefore about 30 ft above ground level.

If R is the distance in feet from the building to ground zero and α is the angle between the vector to ground zero and the normal to the plane of the windows, then the height of burst in feet is

$$H = 30 + R \tan\theta \cos\alpha.$$

The distance of the windows from ground zero was 5040 ft and the angle was 27°. Thus this observation gives the height of burst as 1880 ft.

The same shadows were later measured independently by K. Kimura & E. Tojima (1953) who give their values in one of a series of reports entitled *Collection of investigation reports on the atomic bomb*

disasters, collected by the Japan Science Promotion Society. Kimura & Tojima, from observations on these and other shadows in Hiroshima, give their best average value for the height of burst as 577 ± 20 m or 1890 ± 65 ft. In this paper we shall take the height of burst at Hiroshima as 1890 ft.

Japanese army observers, watching through a good optical system, gave the height of burst at Hiroshima as 550 m (1810 ft) and at Nagasaki 500 m (1650 ft) above the ground.

The three Manhattan District observers calculated from the geometry of the penumbra of the shadows that the diameter of the heat source was 300 ft. However, the calculation is not significant because the meaning of the penumbra and shadow are not the same as in geometrical optics. To cause visible singeing, a threshold of heat is required, and the threshold depends on the rate of deposition of heat. Just inside the shadow (where no singeing occurred) there could have been some heat deposition, and a clear line of sight to part of the fireball, without there being enough heat to cause singeing.

At Nagasaki, no blast damage observations of significance were made near enough to ground zero to require an accurate knowledge of the height of burst. We assume that the height of burst was 1650 ft, the height measured by the Japanese army observers.

3. Blast data and the pressure pulse inside buildings

Blast data

The equations connecting the variables on two sides of a plane shock wave in air are well known and will not be given here. Reference may be made to the book issued by the U.S.A.E.C. entitled *The effects of nuclear weapons*. We follow the notation used in that book and quote a few of the equations relating to blast waves, as well as giving a few simple modifications.

Provided the peak overpressure p is in the region of 5 to 10 lbf/in² the instantaneous overpressure is given closely by

$$p(t) = p(1-t/t_+)\exp(-t/t_+), \tag{1}$$

where t_+ is the duration of the positive phase of the blast. As a matter of observation, as well as from computer calculations, this formula gives a good fit to the blast wave from an explosion in 'free-air'. The formula also, as a matter of observation, gives a good fit for a blast wave in the air near a rigid surface (ground) when there is a single shock front, i.e. at a point swept by the Mach stem. Friedlander (1946) often uses the function (1) in his (linear) theory of sound diffraction and we sometimes call it the Friedlander function.

The density and wind speed being respectively ρ and u, the time variation of the dynamic pressure, $q(t) = \frac{1}{2}\rho u^2$, is related to q, its peak value at the shock front, by

$$q(t) = q(1-t/\tau)^2\exp(-2t/\tau). \tag{2}$$

The relation between (1) and (2) is discussed in the appendix where we use t_+ for the positive duration of the overpressure and τ for the positive duration of the dynamic pressure (the two are not quite the same).

The Mach number M immediately behind the shock front is given by

$$M = 5p/\{7(p_0+p)(7p_0+p)\}^{\frac{1}{2}}, \tag{3}$$

where p_0 is the atmospheric pressure.

The peak dynamic pressure q can be written in the forms

$$q = 5p^2/2(7p_0+p)$$
$$= 0.7(p_0+p)\,M^2. \tag{4}$$

The drag force on a simple symmetrical object in a blast wave is given by

$$F_D = AC_D\,q,$$

where A is the greatest area of cross-section normal to the blast, and C_D is the drag coefficient which must be determined experimentally as a function of the Reynolds and Mach numbers applying to the air flow. We denote the drag force per unit area by

$$S_D = F_D/A = C_D\,q.$$

When an explosion occurs in free air, the variation of the peak shock overpressure and of the positive duration t_+ have been measured experimentally and calculated by a computer from the basic equations. If the explosion were larger it would be necessary to make allowance for the variation of atmospheric pressure and density with height, but in this paper we do not need to make atmospheric altitude corrections.

When an explosion occurs above the ground, and the blast parameters at ground level are required, the situation is much more complicated and full computer solutions do not appear to be available. However, there is a great wealth of experimental data, some of which will be found in the U.S.A.E.C. book quoted above. For explosive (energy) yields comparable with those at Nagasaki and Hiroshima, and for the relevant heights of burst, there is a two-shock configuration of incident and reflected waves out to distances from ground zero about equal to the height of burst. Thereafter, at ground level, the two shocks merge to form a single nearly vertical front called the Mach stem.

The chapter on basic blast parameters given in the U.S.A.E.C. book includes parametric curves in the (H, R)-plane (H is the height of burst and R is distance from ground zero, both scaled to 1 kT), showing peak overpressure and positive phase durations. The ground is assumed horizontal, free of structures and at sea level. However, the physical nature of the surface of the ground affects the blast parameters and modifies the exact shape of the overpressure-time functions. One extreme case is a smooth water surface. The other extreme can be taken as a dry powdery coloured surface, such as might be found in remote areas where nuclear tests have been made.

Depending on the physical characteristics of the surface of the ground, the heat flash from the nuclear explosion causes the air near the ground to be heated and turbulent by the time that the blast wave arrives, producing what is called a thermal layer. In the extreme case of a dry powdery topsoil, and for heights of burst in a certain range, the pressure-time curve near the ground can be substantially different from the 'classical' blast wave. *The effects of nuclear weapons* refers to this effect as 'the precursor'. In other cases, the change of shape of the pressure–time curve from that of the classical blast curve is less marked, but the peak overpressure can in extreme cases be reduced by as much as 20 %.

In the idealized case, geometrical and time scaling laws apply, and small-scale chemical explosions, with carefully controlled conditions, can be used to supplement the data from which the parametric blast curves for nuclear explosions are constructed. However, small scale chemical explosives do not produce a blast wave of exactly the same shape as nuclear explosives nor do they produce a thermal layer. The scaling of the blast from small model chemical explosions to

nuclear explosions must, therefore, be done cautiously, especially in a geometry where the irregular (Mach) reflexion from a rigid surface lies in a sensitive region.

In our opinion, the parametric pressure curves given in the U.S.A.E.C. book are accurate, or nearly so, over most of the (H, R)-plane, but there are errors in the region of the 'knees'. We have some data from a few full-scale nuclear tests over surface conditions where the coupling

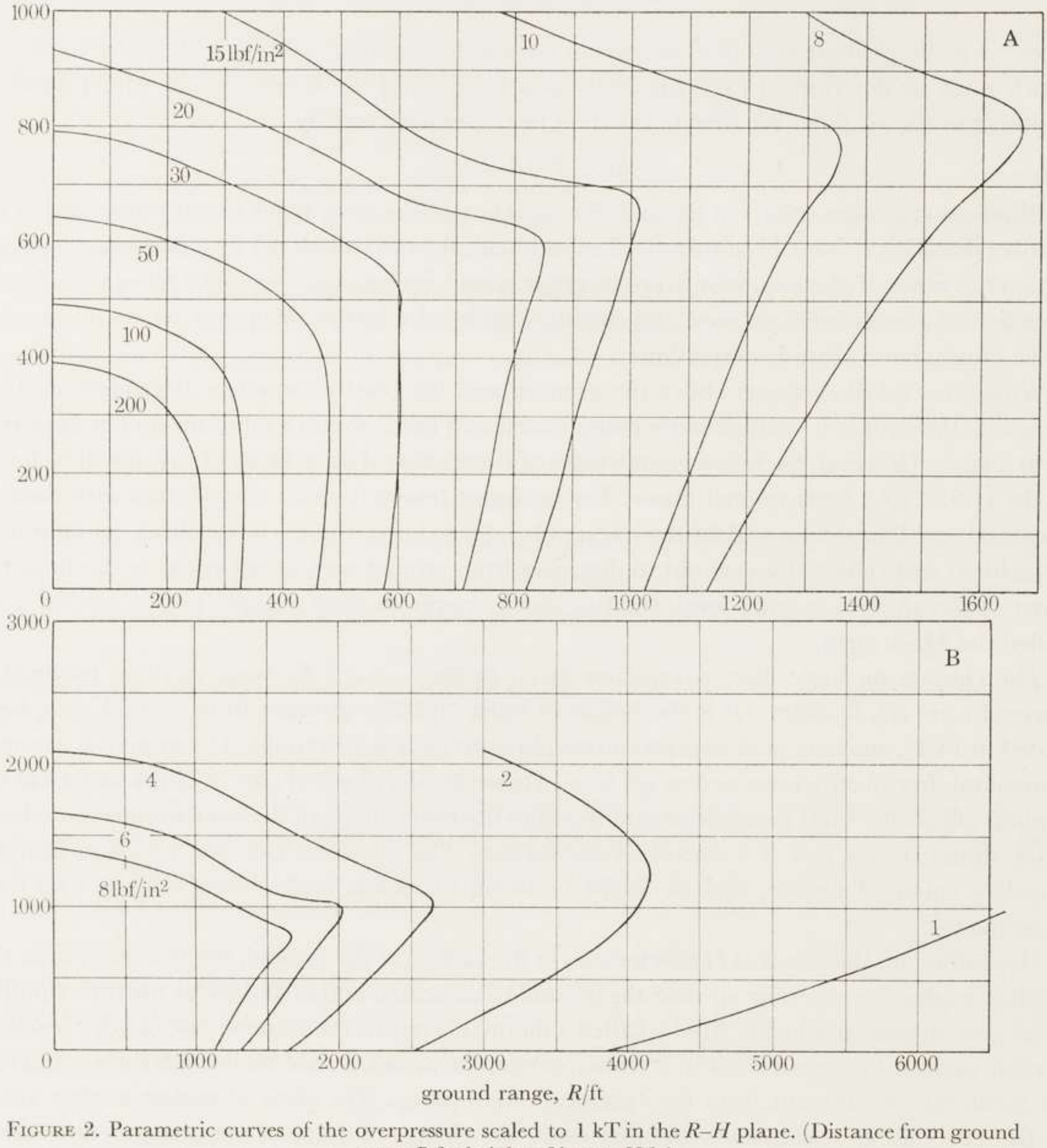

FIGURE 2. Parametric curves of the overpressure scaled to 1 kT in the R–H plane. (Distance from ground zero, R ft; height of burst, H ft.)

between the blast near the ground and the thermal effects was small. We believe that the parametric curves for overpressure given in the book need modification in the region of the 'knees' of the curves for overpressure levels of about 10 lbf/in². Our best estimates, based on the data available to us, are incorporated in figure 2A, B.

The interpretations of the blast observations made in the two Japanese cities depend primarily on the estimated peak overpressures, but in most cases we also require to know with moderate

accuracy the positive phase duration from a scaled unit nuclear explosion. The positive phase duration from a nuclear explosion is not easily measured with high accuracy, but British data agree with the parametric curves given in the U.S.A.E.C. book. The positive duration for the unit explosion is in any case a slowly varying function compared with the peak shock overpressure.

To illustrate the order of magnitude of the corrections we are applying in the small region of the knees of the parametric peak pressure data given in the U.S.A.E.C. book, some results which we have used from certain nuclear tests will be quoted. Observations on poles 10 ft tall showed no thermal layer effect but records taken at ground level did show a disturbed waveform, with a reduction of 10 to 15 % in the peak overpressure. Scaling certain data observations to a distance of 3080 ft from ground zero from a yield of 12.2 kT burst 1890 ft above ground gave the following results. In the case where there was no thermal layer effect, the peak overpressure was 10.3 lbf/in^2, and in the case where there was a marked thermal layer effect (red desert soil) the peak overpressure was 8.4 lbf/in^2. According to the parameter curves given here (figure 2), the overpressure would be 9.8 lbf/in^2 in the conditions considered to exist at Hiroshima and Nagasaki. The U.S.A.E.C. book gives 11.1 lbf/in^2 for 'nearly ideal' surface conditions, i.e. minimal effect of the thermal layer; but this value does not agree with the observations which we have used, and we believe it to be too high.

The calculated response of a drag-sensitive object depends on the shape of the overpressure–time waveform assumed in the calculations. The measured waveforms from nuclear test explosions at comparable heights of burst were therefore subjected to detailed examination. At ground level, for overpressures in the range that we need, the single Friedlander shape given by equation (1) was generally found to be a very good representation, except for situations in the region of the pronounced 'knee' of the overpressure parametric curves. The waveform here was sharply peaked and the shape could be well represented by connecting two expressions, each a section of a Friedlander function

$$\left.\begin{aligned} p &= p_1\phi(\alpha_1 t) \quad (0 \leqslant t \leqslant t_1), \\ p &= p_2\phi(\alpha_2 t) \quad (t_1 \leqslant t \leqslant 1/\alpha_2), \\ \text{where}\qquad \phi(x) &= (1-x)\,\mathrm{e}^{-x}. \end{aligned}\right\} \tag{5}$$

The peak overpressure p_1 and positive phase duration $1/\alpha_2$ are given by the parametric curves already described (figure 2 for p_1 and the U.S.A.E.C. book for t_+ which is now $1/\alpha_2$). It remains to specify the parameters p_2 and α_1 as functions of yield and ground range for the particular heights of burst at Hiroshima and Nagasaki. The values of p_2 and α_1 were obtained for each available nuclear test record, and the yield, distance and time were scaled to correspond to the Hiroshima height of burst of 1890 ft.

Values of the ratios p_1/p_2 and α_1/α_2 were marked against the corresponding data points on two separate graphs of scaled yield against scaled ground range. When contours were drawn between the points on the two graphs, the two sets of lines were found to be similar in shape, and within the accuracy of the measurements a single set of contours could be used to define the two parametric ratios. Further, there was a linear relation between the two ratios on a contour, so that the shape of the pressure–time profile could be characterized by a single 'peaking parameter' k according to the relations

$$\left.\begin{aligned} p_1/p_2 &= 1+\tfrac{1}{5}k, \\ \alpha_1/\alpha_2 &= 1+k. \end{aligned}\right\} \tag{6}$$

The contour plot is shown in figure 3. The cross-over time t_1 can be expressed in terms of k since

$$p_1\,\phi(\alpha_1 t_1) = p_2\,\phi(\alpha_2 t_1). \tag{7}$$

It must be emphasized that these relations apply only within the region covered by the contours in figure 3. The diagram may, however, be scaled to other heights of burst H by varying the ground range as H and the yield as H^3. When this is done for the Nagasaki height of burst of 1650 ft, it is found that the reputed yield of 20 kT would not have given any peaking.

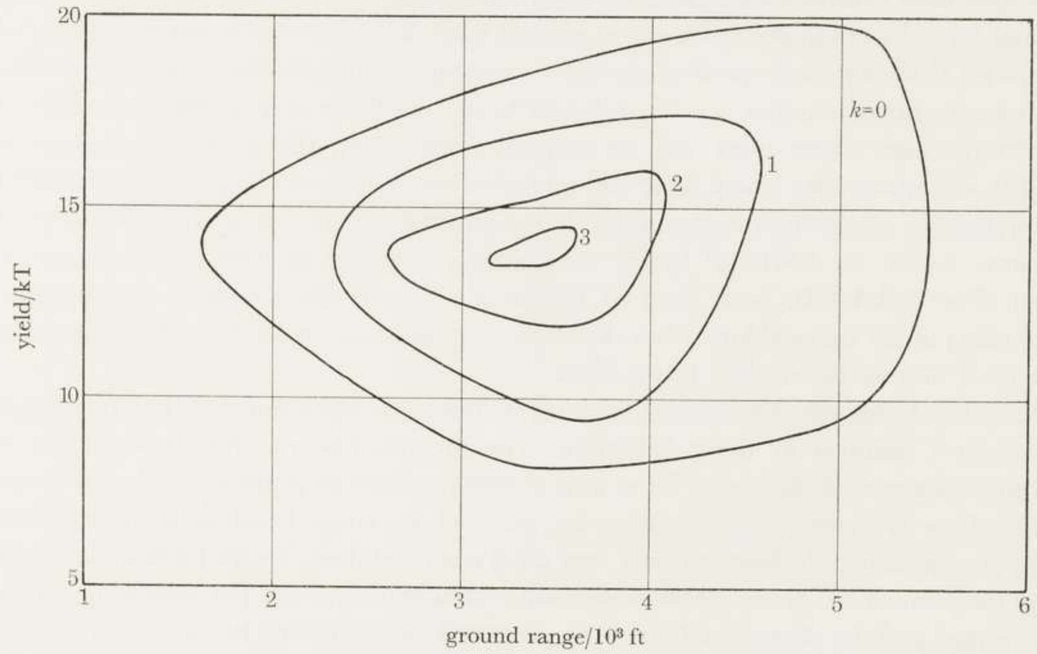

FIGURE 3. The peaking parameter k for various yields with a height of burst 1890 ft.

To interpret some of the observations made in Hiroshima and Nagasaki we shall need numerical values of the peak dynamic pressure q and the positive duration τ of the dynamic pressure (see appendix). When the Mach stem has formed, there is only a single nearly vertical shock near the ground. The calculation of q is simple and involves only the peak overpressure, obtainable from figure 2. We believe that in these cases, the possible error is small. At distances closer to ground zero, the calculations are more difficult arithmetically but the experimental data do not always quite fit to the mechanical equations, at least those of the simplified model which is assumed.

If one assumes that the ambient air is all at the same natural temperature, then the four variables may be taken as p_i, p_r, θ and ϕ; where p_i and p_r are the incident (free air) overpressure and reflected overpressure respectively, and θ, ϕ are the angles of incidence and reflexion. Two mechanical conditions must be satisfied. The speed of travel of the incident shock along the ground must equal the speed of travel of the reflected shock along the ground: and the mass velocity of the air emerging from the reflected shock near the ground must be horizontal. Since the angle θ at any point is fixed by geometry, and the free air pressure is also fixed, the whole system is determinate.

The difficulty is that taking known values of θ and p_i, the values of p_r and presumably ϕ do not agree with observation. We have attributed this to the effect of the warm layer of air near the

ground. Since we do not know enough about the warm layer to make any calculation about its effect, we have to chose a way to proceed. We have chosen two ways which we think bracket the situation.

First, we take q values from the third edition of the U.S.A.E.C. book. The U.S. peak overpressure levels near to ground zero (in the Hiroshima geometry) are somewhat greater than ours and arithmetical mismatch with the mechanical equations is therefore minimal. These q values are thought to be on the high side because the overpressure (according to us) is on the high side.

TABLE 1

R/ft ...	1200	1400	1600	1800	2000	2200
q_{14}(m 1)	2.15	2.60	2.95	3.30	3.70	4.00
q_{14}(m 2)	1.54	1.76	1.92	2.06	2.19	2.30
q_{11}(m 1)	1.35	1.55	1.70	1.85	2.00	2.10
q_{11}(m 2)	1.24	1.43	1.58	1.72	1.84	1.88
τ_{14}	0.67	0.68	0.69	0.69	0.70	0.71
τ_{11}	0.64	0.65	0.66	0.67	0.68	0.69

Units: q, lbf/in^2; τ, second.

TABLE 2

k	0	1	2	3
$\mathscr{I}_k/\mathscr{I}_0$	1.000	0.747	0.584	0.472

Secondly, we take the observed overpressure at any point, use the relationship between the reflexion coefficient, the angle of incidence and the incident pressure to find what the incident pressure should have been. Then we can calculate q. We think that these q values are on the low side because they have too low a value of incident pressure.

Table 1 gives some results for the Hiroshima case with yields of 14 and 11 kT. Values obtained by the first method are denoted by (m 1) and by (m 2) for the second method. The average of the two is probably the best estimate available.

At distances slightly greater than 2200 ft the Mach stem begins to form at ground level, and there is a jump to a new régime with larger values of q. This jump is brought about by mechanical reasons at the shock fronts and they also bring about the peaking in the pulse near the ground.

The time integral of the dynamic pressure over the positive phase of the overpressure, whether the wave is peaked or not, may be called the positive dynamic impulse $\mathscr{I}$. We can calculate by an elementary integration the ratio of the positive dynamic impulse of a peaked wave to that of a wave with the same starting value and the same duration, but having the conventional Friedlander form given by (2). The results are shown in table 2.

The reduction in the positive dynamic impulse is indeed severe for $k = 2$, and even greater for $k = 3$ which corresponds with the maximum in the experimental records. Of course, the peak overpressure is unusually high, and falls at the extreme tip of the knees of the parametric curves of figure 2.

A simple qualitative argument shows that a peaking effect can be expected in certain conditions, and the argument can be put in two related ways. If the angle of incidence to the ground of the spherically expanding wave from the explosion happens to make an angle to the ground which is very close to the angle which gives the sharp maximum in the reflexion coefficient specific to the particular overpressure of the incident shock, the leading edge of the pulse will be

magnified in overpressure by 3.0 to 3.2 (see Fig. 3.71 *b* of the U.S.A.E.C. book). Later parts of the pulse will only be magnified by 2.0 to 2.3, because they are not incident at the angle which maximizes their reflexion coefficient.

The second form of the argument is that in the early stages of the Mach stem, the mass velocity immediately behind the stem is appreciably greater than the velocity above the stem (there is a slip stream). Therefore, at a point near the ground a little beyond where the Mach stem forms, the fluid near the ground coming to the point is not fast enough to sustain the normal rate of decay of overpressure. Peaking must occur.

The above arguments would suggest that the blast waves from chemical explosives should also show peaking.

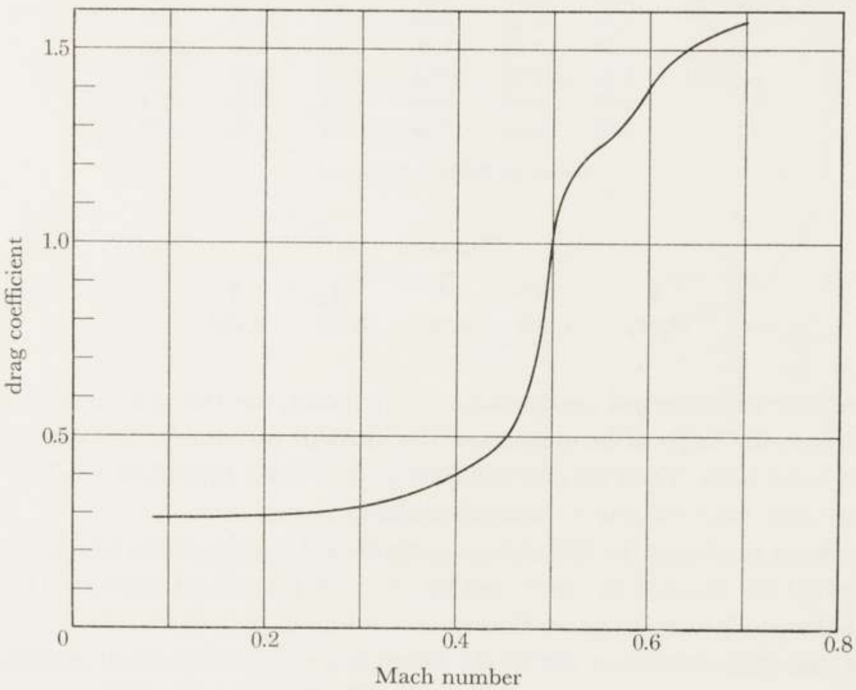

FIGURE 4. The drag coefficient for a long smooth cylinder as a function of Mach number with Reynolds number a few times 10^5 up to about 10^6.

Airburst charges of chemical explosives show the peaking effect but to a much less marked degree. The probable explanation is that nuclear explosives cause the air near the ground to be warmed by heating through the heat flash. In the region where the Mach stem is forming, when the angle of incidence is giving a maximum reflexion coefficient of the shock, a thermal layer would have a strong effect on the extremely sensitive hydrodynamics. Elsewhere, the thermal layer effect would be small.

The drag coefficients

The drag coefficients which we shall need are in some cases independent of Mach and Reynolds numbers in the ranges of interest to us, at least within the limits of experimental error (there are discrepancies of a few parts per cent between different observers using their different facilities). With circular cylinders this simplification does not apply. The Mach number near the front part of the blast wave, the part that does the damage, in our examples is usually between 0.3 and 0.5;

while the Reynolds number is approaching 10^6 in some cases (lightning conductors) and three times greater in other cases (power line poles). The details of the flow pattern passing a long circular cylinder depends on whether laminar separation in the boundary layer has terminated or not. The critical Reynolds number is close to 4×10^5. As the Reynolds number increases through the critical number, the drag coefficient falls rapidly from about 1.2 to 0.3. Provided Reynolds number is above the critical number, but not too far above it, the drag coefficient is a function of Mach number only. We are indebted to the Aerodynamics Department of the Royal Aircraft Establishment for providing us with a curve showing the drag coefficient as a function of Mach number, the Reynolds number being somewhat above the critical number. This curve is shown in figure 4. The book *Fluid-dynamic drag* by Hoerner (1965) gives closely similar data, especially figure 5 of chapt. XV.

We are indebted to the Aerodynamics Division of the National Physical Lbaoratory for providing us with the latest information at higher Reynolds numbers. At Reynolds numbers greater than 10^6 and for Mach numbers greater than about 0.2, the drag coefficient rises linearly in $\lg R$ from 0.35 at 10^6 to about 0.7 at 10^7. The experiments on which this statement is based are mainly those of Roshko (1961), and Cincotta, Jones & Walker (1966). We have to use these data and the information given in figure 4 to make the best choice of the drag coefficient in any particular case.

The pressure pulse inside a building

Suppose that at time $t = 0$, an orifice of area A is opened in a vessel of volume V containing air at pressure p_2 and density ρ_2, there being air outside at pressure p_1 and density ρ_1.

Then the equation of steady stream line flow gives

$$\frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2} = \frac{u^2}{2} + \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1'},$$

where u is the velocity of escape and ρ_1', the density of the air after escape, is given by the adiabatic equation

$$(\rho_1'/\rho_2)^\gamma = p_1/p_2.$$

The rate of scape of mass from inside is

$$-\mathrm{d}M/\mathrm{d}t = Au\rho_1' = Au\rho_2(p_1/p_2)^{1/\gamma}.$$

The rate of change of the density inside is given by

$$V\,\mathrm{d}\rho_2/\mathrm{d}t = \mathrm{d}M/\mathrm{d}t,$$

and the internal pressure and density are related by the adiabatic equation. Thus

$$\mathrm{d}p_2/\mathrm{d}t = (\gamma p_2/V\rho_2)\,\mathrm{d}M/\mathrm{d}t.$$

Substituting for $\mathrm{d}M/\mathrm{d}t$ we obtain

$$\frac{\mathrm{d}p_2}{\mathrm{d}t} = -\gamma\frac{A}{V}p_2\left(\frac{p_1}{p_2}\right)^{1/\gamma}\left[\frac{2\gamma}{\gamma-1}\frac{p_2}{\rho_2}\left\{1-\left(\frac{p_1}{p_2}\right)^{1-1/\gamma}\right\}\right]^{\frac{1}{2}}. \tag{8}$$

If we now consider the inverse case where air is flowing into the vessel, the pressure and density inside and outside being denoted as before by p_2, ρ_2, p_1 and ρ_1, we obtain

$$\frac{\mathrm{d}p_2}{\mathrm{d}t} = \gamma\frac{A}{V}p_2\left(\frac{p_1}{p_2}\right)^{1-1/\gamma}\left[\frac{2\gamma}{\gamma-1}\frac{p_1}{\rho_1}\left\{1-\left(\frac{p_2}{p_1}\right)^{1-1/\gamma}\right\}\right]^{\frac{1}{2}}. \tag{9}$$

First, let us solve the influx equations which would apply when a blast wave envelops a building, there being windows on all sides which break so quickly that they do not impair the entry of the blast. The pressure everywhere inside the main body of the building is assumed the same, and the dynamic pressure effects from the wind of the blast wave are assumed to average to zero. We write

$$p_1 = p_0 + p, \quad p_2 = p_0 + P,$$

where p_0 is the atmospheric pressure, p is the overpressure in the blast wave, and P is the pressure rise inside the building.

We take only first-order terms and linearize the equation to the form

$$\mathrm{d}P/\mathrm{d}t = k(p-P)^{\frac{1}{2}},$$

where $k = (7Ap_0/5V)\,(2/\rho_0)^{\frac{1}{2}}$, and ρ_0 is atmospheric density.

Now write

$$X^2 = p - P, \quad (X \geqslant 0),$$

and obtain

$$\frac{\mathrm{d}p}{\mathrm{d}t} - 2X\frac{\mathrm{d}X}{\mathrm{d}t} = kX.$$

Approximate to $\mathrm{d}p/\mathrm{d}t$ behind the shock front by the linear law

$$p = p_\mathrm{i}(1-2\alpha t), \quad \alpha = 1/t_+.$$

The equation then integrates to give

$$t = (2/k)\{p_\mathrm{i}^{\frac{1}{2}} - (p-P)^{\frac{1}{2}}\} + (4\alpha p_\mathrm{i}/k^2)\ln\left\{\frac{1+f(p-P)^{\frac{1}{2}}}{1+fp_\mathrm{i}^{\frac{1}{2}}}\right\},$$

where $f = k/2\alpha p_\mathrm{i}$.

The pressure inside the building reaches a maximum when $\mathrm{d}P/\mathrm{d}t = 0$, i.e. when $P = p$ and $X = 0$.

The time at which this occurs is given by

$$T = (2/k)\,p_\mathrm{i}^{\frac{1}{2}} - (4\alpha p_\mathrm{i}/k^2)\ln(1+fp_\mathrm{i}^{\frac{1}{2}}), \tag{10}$$

and the pressure inside the building at this time is

$$P_{2\mathrm{m}} = p_0 + p_\mathrm{i}(1-2\alpha T) = p_0 + P_\mathrm{m}. \tag{11}$$

To illustrate the theory, take the following case:

$$p_0 = 10^6\,\mathrm{dyn\,cm^{-2}}, \quad p_\mathrm{i} = p_0/3, \quad t_+ = \tfrac{2}{3}\mathrm{s}, \quad V/A = 2000\,\mathrm{cm}.$$

We find

$$k = 2.828\times10^4, \quad f = 10^{-6}k, \quad T = 41.5\,\mathrm{ms}, \quad P_\mathrm{m}/p_\mathrm{i} = 0.875.$$

Thus, the pressure inside reaches 87.5 % of the initial shock pressure outside, some 40 ms after the blast envelops the building.

The above theory is adequate for most cases of damage to simple systems inside buildings; but if the response time is very short (a few milliseconds) it may be necessary to determine the fine structure of the pressure rise and then make detailed computer calculations of the dynamic response.

The pressure inside a building: effect of window glass

The previous paragraphs have given an approximate theory of the rise of pressure inside a building due to a long blast wave. However, this theory neglects the effect of glass in the window spaces. At high shock pressures, the effect of the glass is relatively less, but at pressures

of a few pounds per square inch, the effect of the glass on delaying the entry of the blast into a building, and on the greatest value in the building, can be estimated to be not negligible.

The following order of magnitude calculation is sufficient for our purposes.

Some windows will be facing the blast, some will be side-on and some will be on the shielded side. We assume that the net inflow through all window spaces is the same as if the air pressure outside the building followed the overpressure in the blast, but that the air was at rest.

The glass shatters almost instantaneously (in not more than 1 ms) and begins to accelerate inwards. A space opens between the window opening and the envelope of the glass fragments, and in channels between the glass fragments.

Suppose that the window dimensions are $a \times b$. Suppose that, at the time t after the start of the motion, the effective size of the opening of each window is θab. Then the force which is accelerating the window fragments is of the order $pab(1-\theta)$, where p is the overpressure of the air outside the building. If m is the mass of the window per unit area, the equation of motion is

$$p(1-\theta) = m\ddot{x},$$

where x is the distance moved by the window fragments.

The relation between x and θ is approximately

$$x = \theta ab/2(a+b) = \theta L \quad (0 \leqslant \theta \leqslant 1).$$

The boundary conditions are $\quad t = 0, \quad \theta = 0, \quad \dot{\theta} = 0.$

The solution is $\quad \theta = 1 - \cos\{t(p/mL)^{\frac{1}{2}}\}.$

The choking effect of the window glass is zero after time

$$t_1 = \tfrac{1}{2}\pi(mL/p)^{\frac{1}{2}}. \tag{12}$$

The effect of the 'choke' on the influx of air is the same as if the choke were complete up to time $(mL/p)^{\frac{1}{2}}$, and were zero thereafter.

In the case of the Hiroshima Communications Bureau, p was about 5 lbf/in^2, m was not more than 1 g/cm^2 and L was about 30 cm. The effect of the glass was equivalent to a complete choke for a time not more than 10 ms and zero thereafter.

4. Drag observations on circular cylinders

Observations were made in the two cities on a number of cylindrical poles where it seemed likely that a significant estimate could be made of the blast wind. Telegraph poles and power line poles had snapped in both cities, and the three Manhattan District observers made measurements where the damage effects appeared to be delimiting. In Hiroshima, where many large buildings were within about 3000 ft of ground zero, lightning conductors, some also used as flagpoles, on the roofs of these buildings, were bent by the blast. A few examples were found where the angle of yield was 5 to 10° and these were measured. The most promising case was a pair of identical mild steel tubular poles on the roof of the Hypothec Bank in Hiroshima, and a part of one of these poles was taken to England for test. A similar situation was found on the roof of the Chugoku Building.

In this section, we interpret these various observations with the aid of basic data given in § 3.

Poles on Hypothec Bank and Chugoku Building

Two identical mild steel cylindical lightning conductors, also used as flagpoles, on the roof of the Hypothec Bank of Japan, 3080 ft from ground zero at Hiroshima, were bent by the blast. Figures 5, 6 and 7, plate 2, show photographs of the building and of the poles. One pole was cut 1 in above the stone support and a 46 in length was taken to England for test.

The outside diameter of the pole was 2.40 in and the thickness of the metal was 0.140 in. At the top of the pole was fixed a light metal sphere which carried a lightning conductor spike. The free length of the pole from the surface of the stone support to the top of the ball was 239 in. In our calculations, we take the free length of the pole as 239 in, and ignore the spike and make no correction for the presence of the sphere.

The angle between the straight part of the pole and the vertical was $6\frac{1}{2} \pm \frac{1}{2}°$, i.e. 0·144 rad. The angle for the other pole was the same within $\frac{1}{2}°$.

Tests were made at the National Physical Laboratory in November 1945 by Dr G. A. Hankins to measure the bending moment which would cause the pole to yield. A close examination of the 46 in length showed that some distortion could be detected up to about 18 in from the end near the support (i.e. 19 in from the surface of the stone support). A straight part of the length was cut, and strain gauges were fixed around a section where the strain was to be greatest. The specimen was put in a testing machine, using four-point loading, and a bending moment was applied. The strain gauge readings were plotted as a function of bending moment, and the bending moment at which the maximum strain recorded by a gauge began to depart from linearity in the applied bending moment was determined. The bending moment which took a cross-section to the limit of the linear relation was 29 200 lbf in.

The results of these tests show that, in the actual motion of the two poles at Hiroshima, the greatest value of the bending moment during the motion at 19 in above the plane of the surface of the stone support was 29 200 lbf in. This result may be regarded as a boundary condition in our theoretical analysis.

There were two similar mild steel poles, each carrying a cable to a ball and spike at the top of the pole, on the roof of the Chugoku Electric Company Building 2200 ft south of ground zero in Hiroshima. The poles also had collars with rings held to the pole by small screws, presumably to take the ropes of a flag. Both poles were bent by the blast. One of the poles was mounted near the cupola on top of the rectangular superstructure above the stairs leading to the roof on the south side of the building. Figure 8, plate 3, shows a photograph of this pole. The other pole was 26 ft south of the south edge of the roof facing the blast, and was mounted at the mid-point of a low parapet wall over the top of the auditorium. This was the pole that was measured, since it was thought that the wind flow around the other pole would be greatly disturbed by the cupola. The free length of the pole above the rim of the support, to the mid-point of the ball, was 114 in. The material and the cross-section were the same as those of the two poles on the Hypothec Bank, but no specimen was taken for later examination in the laboratory. The angle of set of the pole was 4.3° or 0.075 rad and the distance of the pole from ground zero was 2190 ft. The other pole, near the cupola, was more severely bent and the bend was 16°. We allow for the drag effects on the ball and arms by assuming that they were equivalent to an extra length of pole. The free length was therefore taken as 118.5 in.

The motions of the poles are complicated dynamical problems. The poles were flexible and their longest natural periods were neither long nor short compared with the positive duration of

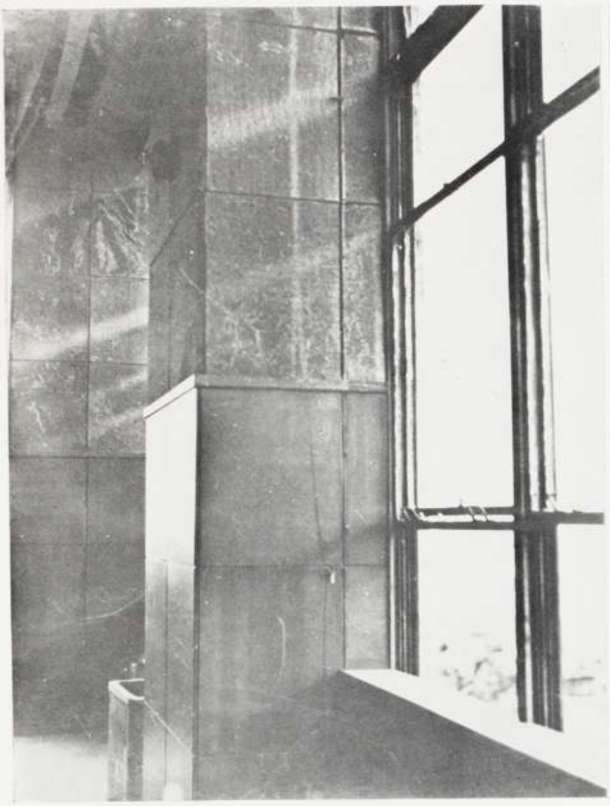

FIGURE 1. Beaver board wall surface singed by the heat flash at Hiroshima. The geometry of the shadows gave the height of burst.

FIGURE 6.

FIGURE 5.

FIGURE 7.

FIGURES 5 to 7. The Hypothec Bank in Hiroshima on the roof of which two similar lightning conductor poles were bent by the blast.

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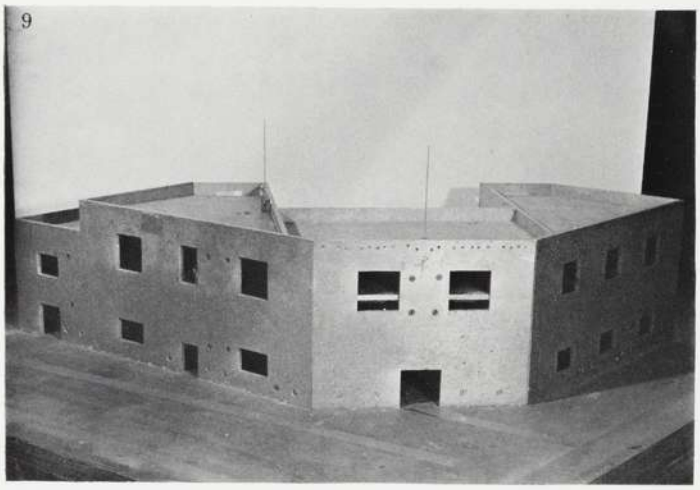
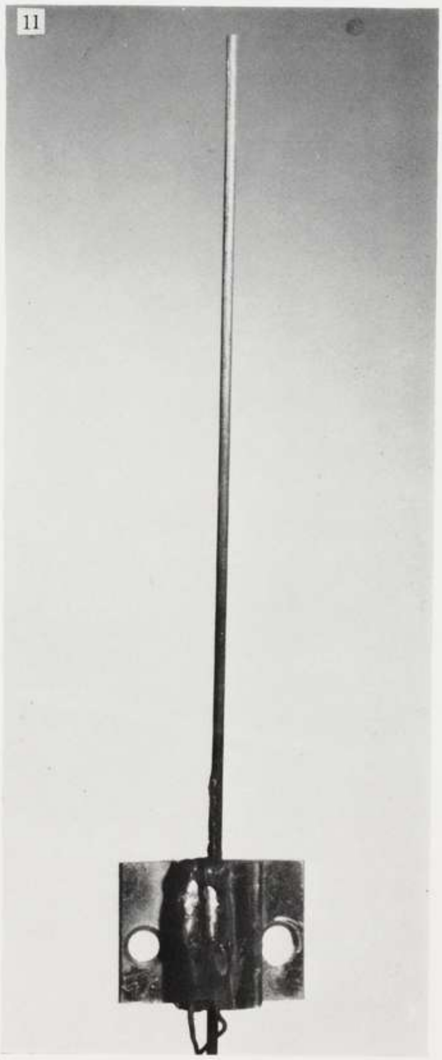
FIGURE 8. The bent lightning conductor pole near the cupola on the roof of the Chugoku Electric Company building in Hiroshima.





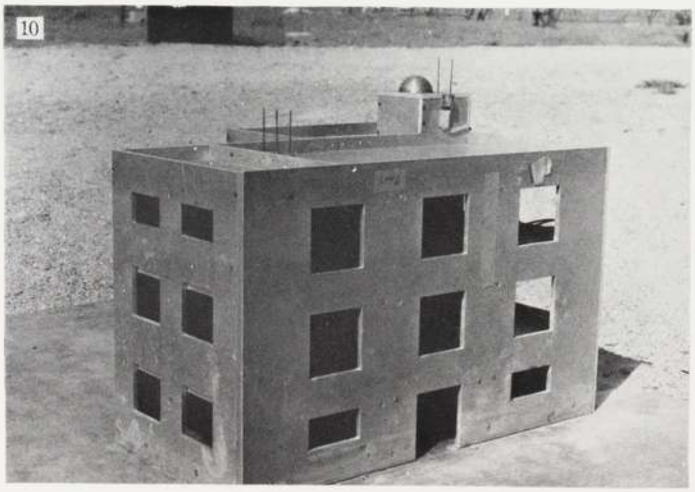


FIGURE 9. Model of Hypothec Bank for field tests on the poles.

FIGURE 10. Model of Chugoku Electric Company building.

FIGURE 11. Mounting of the strain gauges on the $\frac{1}{24}$ scale model poles.

FIGURE 14. A power line pole which did not break, an I beam power line support pole which bent and an uprooted tree near the Appeal Courts in Hiroshima.

FIGURE 17. The city offices in Hiroshima.

FIGURE 15. Overturned and not-overturned memorial stones near the Kokyo Temple in Hiroshima.

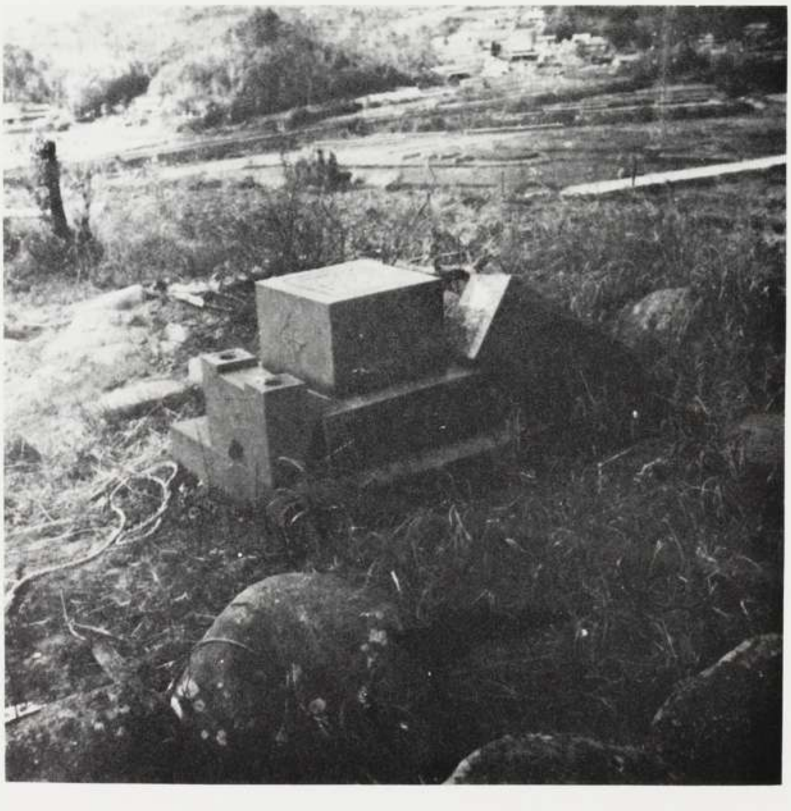


FIGURE 16. Overturned memorial stone in Nagasaki.

the blast. It is not obvious without elaborate calculations whether the peak blast winds gave drags which were appreciably greater, or approximately equal to, or appreciably less than the gradually applied load which would have caused failure. The poles were on buildings and the blast winds were greater than what they would have been if the buildings had not been there. Classical hydrodynamics suggest wind speed enhancements perhaps 50 %. The pressure measurements on the top of rectangular prisms discussed in § 5 establish that there is a suction over the roof, relative to the hydrostatic pressure prevailing in the absence of the prism (or building), of some 80 % of the dynamic pressure. From this, it may be inferred that the wind speeds over and close to the roof have been enhanced by an amount of the order of 30 %.

Finally, one of the sites (the Hypothec Bank) happened to be very close to the position where our basic data on nuclear explosions show a maximum peaking effect at ground level.

We decided that significant calculations could not be made until we had made model experiments to determine the enhancement of the drag acting on the poles, due to the building structures.

Model experiments

After certain preliminary tests we decided to work on the scale of 1/54.

The external dimensions of the two buildings were reduced by this scale factor, and models were constructed from aluminium alloy plate $\frac{1}{4}$ in thick. The distribution of the windows was simplified to facilitate construction of the models, but the correct aperture ratio was retained for each wall. Photographs of the models are shown in figures 9 and 10, plate 3. The geometry of the models was considered sufficiently realistic to establish the correct pattern of reflected and diffracted waves at the locations of the poles.

Models of the poles, instrumented with strain gauges, were mounted in the correct places on the model buildings, and were also mounted on flat boards at ground level at the same distances from the model ground zero.

The gravest mode of the real poles on the Hypothec Bank by calculation had a period 0.632 s, and the gravest mode of the model pole, measured from the strain gauge records, was 11.7 ms, exactly in the scale 1/54. Similarly, the poles on the Chugoku Building had a gravest period of 0.152 s, and this was modelled on the scale of 1/54. The average diameter of the model poles, by measurement, was 0.063 in. The poles were supported by setting them into epoxy resin within a brass ring which was attached by screws to the model building. Details of the mounting are shown in figure 11, plate 3. The poles which were exposed at ground level were similarly mounted on a steel bar which was screwed into a slot in a wooden board 3 ft × 3 ft × 2 in. The board was bedded on to sand.

The material of the model poles was not stressed beyond its yield point at any time during the motion. The bending moment at the point of support was measured by means of a silicon strain gauge, Ferranti type Z PG 12 P, attached by epoxy resin. This was 0.3 in long by 0.01 in wide and was mounted longitudinally on the model pole with its centre approximately 0.2 in above the point of support. Because of the small size of the model poles it was impossible to locate the strain gauges very precisely, so each pole was calibrated by mounting it horizontally and hanging a series of weights on the free end.

The blast wave was produced by bare spherical charges of RDX/TNT, 60/40 (composition B) weighing 64 lb each. The charges comprised two cast and machined hemispheres cemented together and were centrally initiated by a no. 8 electric detonator. The charges were suspended in a nylon net from a cable slung between two masts 48 ft high. The height of the centre of the

charge above the ground was 35 ft (corresponding to 1890 ft full scale). The position of the charge was adjusted by guy lines with an accuracy of at least 3 in (half the charge radius).

The overpressure–time variation was measured side-on by quartz piezoelectric transducers mounted respectively flush with the ground and in a streamlined baffle at the height of the model poles on the buildings.

A separate experiment in a microscale range obtained a Schlieren photograph of the shock wave geometry around the Hypothec Bank. At roof level, there was a two-shock pattern but at ground level there was a single Mach stem. For the Chugoku Electric Company Building, there were two shocks at roof level and also at ground level. The building was in the region of regular reflexion.

TABLE 3. PEAK OVERPRESSURE MEASURED IN MODEL TESTS

	overpressure/lbf in^{-2}		
	round no. 1	round no. 2	mean
Chugoku Building			
ground baffle	10.8	10.8	10.8
roof level	8.4	8.6	8.5
Hypothec Bank			
ground baffle	8.6, 10.0	8.7, 8.8	9.0
roof level	6.4	6.6	6.5

TABLE 4. MAXIMUM BENDING MOMENT MEASURED IN MODEL TESTS

	bending moment/lbf in		
	round no. 1	round no. 2	mean
Chugoku Building			
poles at ground level	0.40, 0.52	0.48, 0.52	0.48
poles on roof	0.64	0.66, 0.71, 0.72	0.68
Hypothec Bank			
poles at ground level	0.68, 0.87	not recorded	0.78
poles on roof	0.80, 1.03	1.00	0.94

We present the results obtained from two charge firings, designated rounds 1 and 2. The peak overpressure values recorded by the pressure transducers are given in table 3, and the maximum values of bending moments indicated by the strain gauges on the model poles are given in table 4. These results indicate that the loading on the poles of the Hypothec Bank was greater than would have been experienced by similar poles on flat ground, the ratio of bending moments being 1.2 ± 0.1; and the shorter pole on the Chugoku Electric Company experienced a considerably greater moment than it would have done at ground level, the ratio being 1.4 ± 0.1. The limits of uncertainty assigned to these ratios were estimated by considering the form of the strain records which are shown in figures 12 and 13. The general form of the response is consistent from one firing to the other, but there is some variation in the amplitude and phase of the higher frequency modes. There was also some breakthrough of the electrical supply frequency of 50 c/s. This is not apparent to the eye, but in some cases it amounted to 10 % of the signal amplitude.

The experimental records of the overpressure–time at ground level at the site of the model Chugoku Building were of perfect Friedlander shape with a peak value of 10.8 lbf/in^2 and a positive duration of 9.5 ms, corresponding on the full scale to 0.51 s. These parameters correspond in the same geometry with a nuclear explosion of 11 kT. Figure 3 shows that such a nuclear explosion would not exhibit the peaking effect at the observation site. Let us therefore scale our

bending moment results on the model poles to see if we get acceptable values. All of the model poles in our experiments were in conditions where the drag coefficient was 1.0; and the drag on the model poles on the Chugoku Building was 1.40 times the drag on the corresponding poles at ground level. Therefore the dynamic pressures were in the same ratio. However, the air density

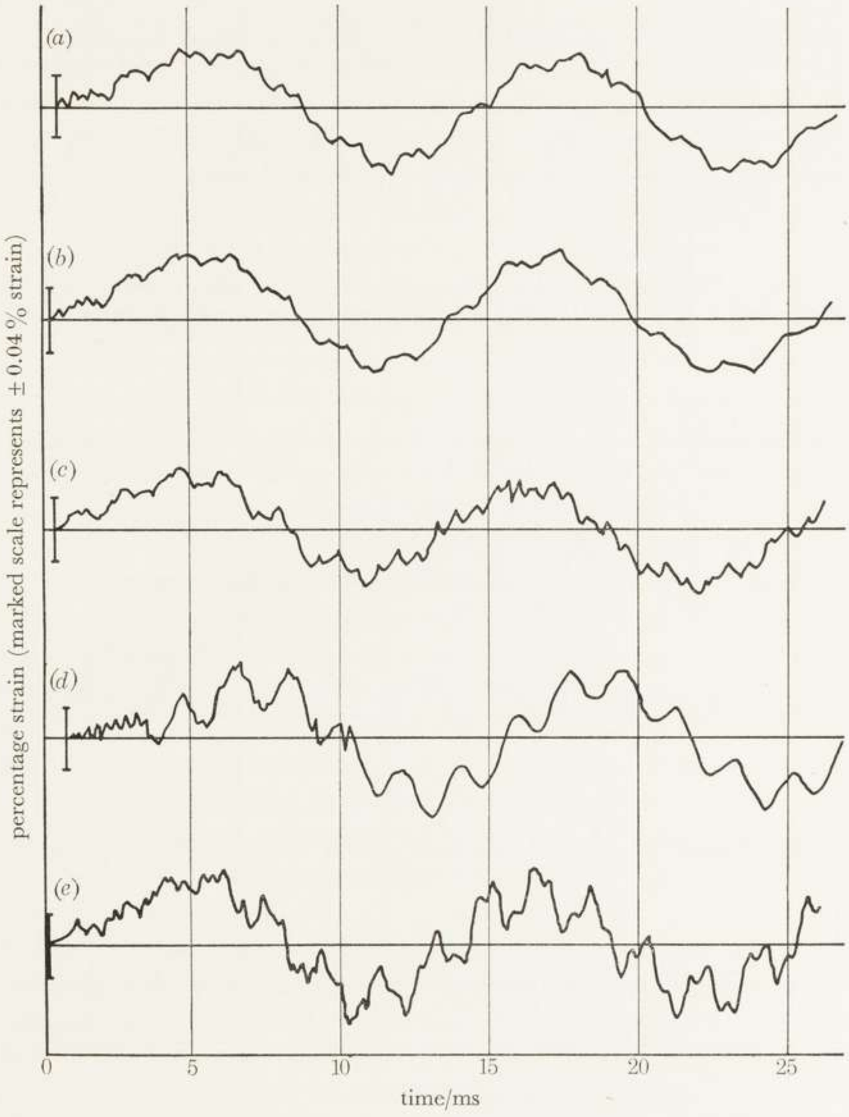


FIGURE 12. Records from strain gauges on the model poles on the model Hypothec Bank. (*a*) Round 10; ground level; 0.113 % strain $\mathrm{lbf^{-1}\,in^{-1}}$. (*b*) Round 10; ground level; 0.100 % strain $\mathrm{lbf^{-1}\,in^{-1}}$. (*c*) Round 10; roof; 0.100 % strain $\mathrm{lbf^{-1}\,in^{-1}}$. (*d*) Round 10; roof; 0.096 % strain $\mathrm{lbf^{-1}\,in^{-1}}$. (*e*) Round 10; roof; 0.100 % $\mathrm{lbf^{-1}\,in^{-1}}$.

around the poles on the building was a few parts per cent less than that near the corresponding poles on the ground. Hence the Mach number for the flow around the model poles on the building was 20 % greater than the Mach number for the poles on the ground, at corresponding times, and the same result would have applied on the full scale.

Thus, in the model experiments, the Mach number of the air flow around the poles started at about 0.3 increased by 20 %, but the Mach number decreased during the motion. We may take

the average drag coefficient as 0.33. Then our model results show that if the full scale pole on the building were behaving elastically, the maximum bending moment would have been M_2, where

$$M_2 = M_1 \times 54^2 \times (2.40/0.63)\,(0.33/1.0),$$

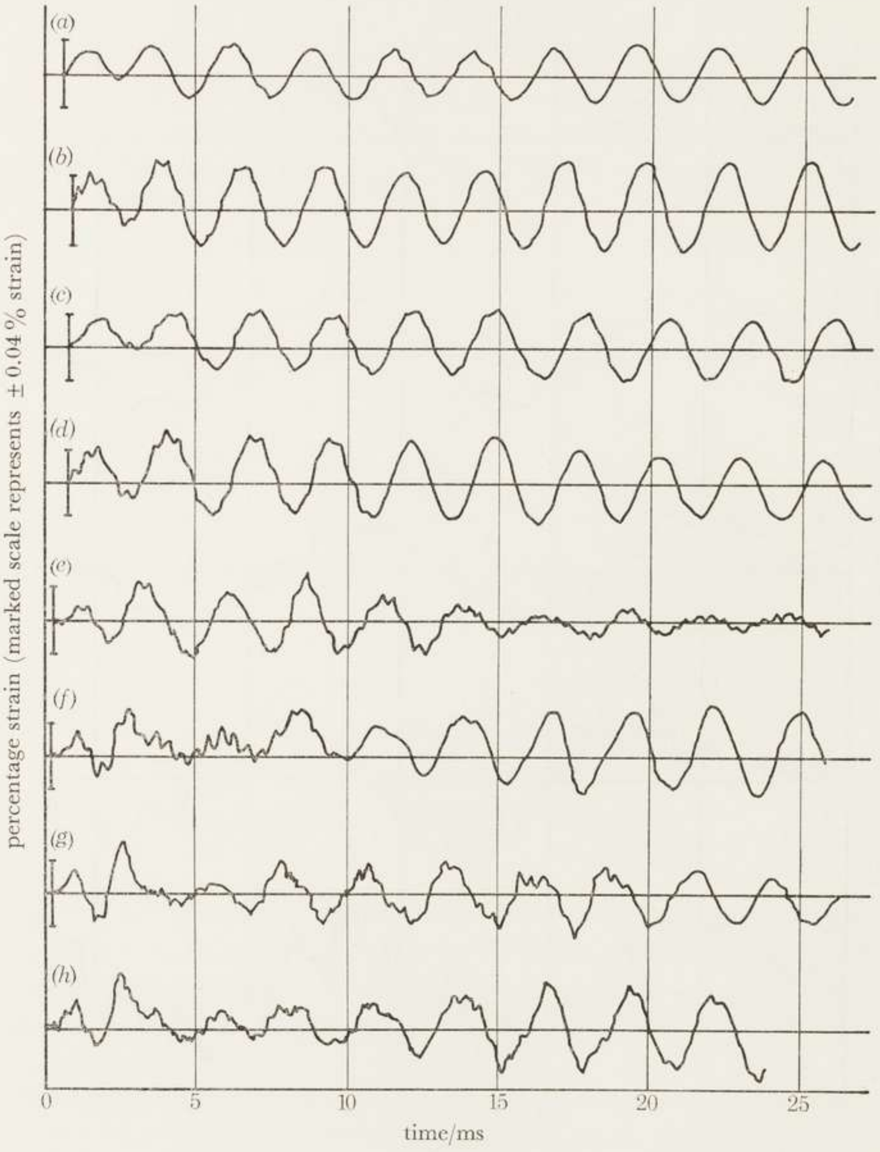


FIGURE 13. Records from the strain gauges on the model poles on the model Chugoku Electric Company building. (*a*) Round 10; ground level; 0.100 % strain $\mathrm{lbf^{-1}\,in^{-1}}$. (*b*) Round 10; ground level; 0.117 % strain $\mathrm{lbf^{-1}\,in^{-1}}$. (*c*) Round 12; ground level; 0.100 % strain $\mathrm{lbf^{-1}\,in^{-1}}$. (*d*) Round 12; ground level; 0.117 % strain $\mathrm{lbf^{-1}\,in^{-1}}$. (*e*) Round 10; roof; 0.094 % strain $\mathrm{lbf^{-1}\,in^{-1}}$. (*f*) Round 12; roof; 0.104 % strain $\mathrm{lbf^{-1}\,in^{-1}}$. (*g*) Round 12; roof; 0.091 % strain $\mathrm{lbf^{-1}\,in^{-1}}$. (*h*) Round 12; roof; 0.097 % strain $\mathrm{lbf^{-1}\,in^{-1}}$.

where M_1 is the observed maximum bending moment in the model experiments (0.68 lbf in). This formula gives M_2 as 25 000 lbf in; only 76 % of the elastic limit. Obviously, a substantial increase in the drag would be required to give a set of $4\frac{1}{2}°$ in the full-scale pole.

A similar calculation for the poles on the Hypothec Bank, using the observed 0.95 lbf in as the maximum bending moment in the model, gives 34 500 lbf in as the maximum bending moment

in the full scale poles on the roof. This is 5 % above the elastic limit, and would therefore give a small angle of set. However, a nuclear explosion of 11 kT is slightly peaked at ground level at the site of the Hypothec Bank, and it is not obvious now much greater nuclear explosion yield than 11 kT would be required to give the observed set of $6\frac{1}{2}°$. We look at this question in the following discussion.

Computer calculations

We need the equations of motion and the boundary conditions.

In principle we are dealing with a system with four independent variables (three space and one time). Starting with fundamentals, we are given a time-independent stress-strain relation for the material. The relation is of classical form. There is linearity up to a certain stress; thereafter the strain increases more rapidly with the stress, with the stress having a maximum possible value (in the static case) perhaps 30 % greater than the elastic limit.

As the blast reaches the pole, the motion begins elastically, and classical elasticity theory (in particular St Venant's principle) gives us the formal solution except for the details very near the clamp. The motion proceeds further, and at one point in the pole (probably the leading edge at the clamp facing the blast) the elastic limit is passed. The stress at this point no longer is linear in the strain, and from this point as the pole continues its motion, a surface grows in the material inside which the elastic limit has been passed. This surface has a complicated shape: it is not horizontal, but extends higher at the leading edge facing the blast, and on the reverse side, than it does in the transverse direction. The division of the material into the one part that is still elastic and the other part where the elastic limit has been exceeded, varies with time. The boundary conditions that are clearly defined by the experimental observations are the final shape of the bend, the angle of set (angle between the vertical and the final straight part of the pole) and the fact that the elastic limit was just reached at a point 19 in above the clamp.

Now a calculation with four independent variables and several régimes is too ambitious for our purposes, and we must make the kind of approximations which give good results for ordinary beam calculations. We use an approximation which may be considered as an extension of St Venant's principle, and reduce the independent variables to two, namely, a coordinate along the central axis of the pole and time.

The model that we propose to use is an elastic pole of length 239 in held by a plastic hinge. So long as the bending moment is less than a certain quantity M_0, the hinge does not yield. When the hinge is yielding, it provides a bending moment resistance of M_0. We assume that the magnitude of M_0 does not change during the motion of the pole. The equation of motion of the model is discussed in the appendix. A similar model is used for the shorter pole.

Clearly M_0 is greater than the elastic limit of the bending moment, 29 200 lbf in, measured experimentally on a length of the actual pole. The best value for M_0 is an average value between the elastic limit value and some higher value, obtained from measurements on bending the pole, or from some theory based on the stress–strain relationship of the material.

There are various arguments we can use to calculate the best value to take for M_0. First we could argue from the case of a uniformly loaded cantilever of length 239 in. If the bending moment 19 in from the clamp is 29 200 lbf in the bending moment at the clamp is 18 % higher. Then the average value of M_0 could be taken as the average of 29 200 lbf in, the value when the elastic limit at the clamp is reached, and a value 18 % higher, the estimated maximum value. This gives a value of 31 800 lbf in, 9 % higher than the elastic limit.

We can make a similar argument based on the bending moment at the clamp at the stationary position in the fundamental mode of oscillation. This argument gives a value for M_0 of 30 800 lbf in, 5.5 % higher than the elastic limit.

Our preferred method of estimating the best value of M_0 was experimental. We have loaded statically some mild steel tubes with the correct ratio of outside diameter to thickness to give a number of angles of set. Taking the angle of set as the independent variable, the average bending moment at the clamp between the elastic limit at the clamp and a set of $6\frac{1}{2}^\circ$ was measured to be 12.2 % greater than the bending moment at the elastic limit. The corresponding figure for a set of 4.3° (the angle of set of the similar lightning conductors given in the next observations) was 11.5 %. Since the two values are very close together, and the difference is less than the possible error in either quantity, we take M_0 as 12 % greater than 29 200 lbf in, i.e. 32 800 lbf in both for the poles on the Hypothec Bank and on the Chugoku Electric Company Building.

By 'lumping' the set into one point at the plastic hinge, rather than having a growing region of set near the hinge, we shall make the position of the elements of the pole during the motion slightly different from their true position. Measurements on the photographs of the poles show that, in the final position of rest, the horizontal displacement of the top of the pole would be calculated as 1 in greater than the true displacement of 27 in. Since the dynamic effects are small compared with the plastic yield terms, the error due to the slightly incorrect displacement of the elements of the pole is negligible.

It is interesting to note that the value of M_0, the 'average' value of the yielding bending moment at the clamp, over a yield of a few degrees in the pole just above the clamp, is in good agreement with a very simple calculation of the yielding bending moment, based on an idealized model. If one assumes that, during yield, the stress on one side of the central (neutral) plane is everywhere the yield stress in tension, S_y, and on the other side is everywhere the yield stress in compression, also S_y, then as far as bending moment is concerned the details of the non-physical discontinuity of stress at the middle does not matter, and the bending moment is

$$M_0 = \tfrac{4}{3}S_y(a^3 - b^3),$$

where a and b are the external and internal radii. Substituting for M_0, a and b, it will be found that S_y is 45000 lbf in^{-2}, a value commonly quoted as about the maximum for the 0.2 % yield proof stress of mild steel.

The first calculations we made on the computer were to obtain the responses of various poles subjected to a load having the time variation of the dynamic pressure given by equation (2). There are effectively two times involved in any calculation—the positive duration τ of the dynamic pressure and the period of the gravest mode T. For each value of the ratio τ/T we calculated the maximum bending moment at the foot of the pole during the motion, as a multiple $\mathscr{M}$ of the bending moment which would result from the steady loading by the starting peak load. Table 5 shows some of our results which can be used by interpolation for the various poles discussed in this section.

Table 5

τ/T	0.97	1.27	1.94	2.47	3.47
$\mathscr{M}$	0.87	0.97	1.16	1.29	1.37

The pulse was peaked at ground level at the site of the Hypothec Bank but the poles were well above the slip stream and the pulse there was not peaked. The shape had been determined by reflexion of the free air pulse at the ground much nearer to ground zero, where there was no

peaking. It is a fair inference from our measurements of the pulse shape on poles and at ground level, that pulse shape changes near the ground do not quickly pass to higher levels.

The model experiments showed that the dynamic pressure on the poles on the roof of the Hypothec Bank were 20% greater than that on poles on the ground, although the latter were in the air flow of the Mach stem of a blast wave with simple Friedlander shape. The enhancement effect of the building on the air flow past the poles was appreciably greater than 10%; it was an enhancement of 10% on the faster flow near the ground. The fact that the actual nuclear pulse at ground level was peaked is of no consequence in this calculation.

Our computer solutions show that a nuclear explosion of 10.0 kT would give the observed angle of set 0.114 rads for the poles on the Hypothec Bank.

Turning now to the poles on the Chugoku Electric Company Building, we know that there was a two-shock configuration at roof and at ground level. (Explosions greater than 16 kT would have formed a Mach stem near the ground.) The most sensitive function in the basic data is the drag coefficient. It is obvious that the stresses generated at Hiroshima in the poles on the Chugoku building were about the same as those generated in the poles on the Hypothec Bank; whereas our model experiments showed that in the model conditions, the bending moment on the poles on the roof of the bank were 40% the greater. The only explanation we can see is that the poles on the Choguko building were in conditions where the drag coefficient was greater than 0.5. This idea is reinforced by the fact that the pole near the cupola of the Chugoku building was bent 16°, and to bend the pole by this amount would have required a much more powerful drag than that which acted on the other pole (nearly half as much again). Our model experiments showed that the drag on the pole near the cupola might have been 5% or so greater than that on the other pole. Both poles must have been in a blast which began with a high drag coefficient, but drag on the pole near the cupola was much the more severe because the drag coefficient was extremely sensitive to Mach number. According to figure 4, the drag coefficient rises steeply for Mach numbers between 0.45 and 0.50. The exact position of this very steep rise in the drag coefficient is crucial for our calculations on these observations. Gowan & Perkins (1953) quote data from the 8 ft and the 16 ft Langley wind tunnels on a 3 in diameter cylinder from Mach 0.2 to Mach 0.6, and these are practically identical with our conditions. There is a vertical rise in C_D at Mach 0.40 from 0.3 to 1.0.

Allowing for the 20% enhancement in the Mach number due to the presence of the building, the observations on the poles show that, according to figure 4, the peak Mach number over level ground would have been 0.42; and according to the data of Gowan & Perkins would have been 0.36; corresponding with peak dynamic pressures of 3.8 and 2.5 lbf/in^2. The computer calculations show that a peak drag force of 1.70 lbf/in^2 with a duration of 0.60 s, with shape given by equation (2), would have given the observed set. So the observations on these poles show that the peak dynamic pressure in the absence of the building was slightly less than 3.8 or 2.5 lbf/in^2 according to which set of drag data is used. If we take the average of the two and compare with the average of method 1 and method 2 in table 1, the estimated explosive yield is 14 kT; but the possible error is at least $\pm$ 4 kT.

The poles on the Chugoku building happened to be at a site where the basic blast and drag data are not known well and are very sensitive to the precise values of the independent variables.

Wooden power line poles

There were many snapped power line poles and telegraph poles in Nagasaki, and many snapped telegraph poles in Hiroshima. The three Manhattan District observers at first thought that it would be possible to draw contour lines in the two cities limiting the critical distances at which certain types of poles were snapped or were not snapped. They soon found that there was too much variability between poles of different lengths, and between their loading by wires and cross-arms, to make this productive. Another method was then tried. An attempt was made to find poles carrying a minimum of cross arms and wires where the general appearance of poles in the neighbourhood indicated that a particular pole must have been near the point of failure in the blast. When such a pole was found the dimensions were measured.

Table 6. Nagasaki

distance from ground zero/ft	place	dimensions	
3300	single power spur line south east of Torpedo Works	diameter at base	8 in
		diameter at top	7 in
		height above ground	27 ft
2500 (two similar poles)	these poles were on the west side of the river in Takenokubomachi; single power line giving no support to poles	diameter at base	8 in
		diameter at top	6 in
		height above ground	17 ft
1800		diameter at base	12 in
		diameter at top	8.5 in
		height above ground	26 ft
1500		diameter at base	8 in
		diameter at top	6 in
		height above ground	17 ft

Generally speaking, the poles carrying telephone wires were tall and carried many wires on the cross-arms. A great many such poles, 30 to 35 ft tall and 12 to 16 in base diameter, were snapped in the two cities. The surfaces of these poles had usually been stained brown by preservatives, probably creosote, and the texture of the surface had many splinters. On the other hand, the power line poles were usually smaller and the natural surface was nearly white and smooth.

The most significant observations on power lines in Nagasaki are described above. In every case, the wood of the pole was in excellent condition and the stump of the pole was still tightly fixed in the ground. Observations on the other poles are not recorded here because they lead to lower values for the least value of the explosive yield, and the observations are therefore less significant than those recorded in table 6.

There were many different poles along the main streets of Hiroshima carrying wires, and once again the simplest type for study appeared to be a set of poles $25\frac{1}{2}$ ft tall, of the same smooth white wood, apparently identical with those measured in Nagasaki. A number of main streets in the centre of the city had some poles of this type to carry wires across the street from which the tramcar lines were hung. These poles were all similar. A search along the main tram routes radiating from the area containing ground zero failed to find any of these particular poles which had snapped, although a few had turned in the ground. On the other hand, quite substantial poles carrying many wires, and with large cross-beams, had snapped up to a distance exceeding 3000 ft from ground zero. Many of the light steel framework structures used as poles carrying overhead wires had also failed badly. Some of the street car wires were supported by cross-wires

strung from rolled steel joists of I cross-section, set vertically in the ground, and some of these joists had bent 10 to 20° or more and a few were prostrate (see later). Figure 14, plate 4, shows a photograph of a power line pole which did not fail, a rolled steel joist pole which did fail and a large uprooted tree in Hiroshima, near the Appeals Court, 1300 ft SSE of ground zero.

The dimensions of the power line pole, measured from its shadows, in Hiroshima and shown in figure 14 were:

height of pole above ground	25 ft 6 in
diameter of pole at base	6.8 in
diameter of pole at top	6.1 in
distance from ground zero	1300 ft

It was closely duplicated by many more in the same street. None of these had snapped. We can therefore use the dimensions as given to estimate the upper limit to the yield of the Hiroshima explosion from the observations that no pole of these dimensions was snapped at any distance.

A sample of the wood of the snapped pole found in Nagasaki at 3300 ft from ground zero was taken to England and was identified by the Forest Products Research Laboratory, Princes Risborough, as either *Thuyopsis dolobrata* or as *Conninghamia lanceolata*. According to the Laboratory, the tension which would cause the fibres at the surface to snap under bending, when the moisture content was 12 %, would be 9840 lbf/in^2 for the *T. dolobrata* and 7160 lbf/in^2 for the *C. lanceolata*. The corresponding moduli of elasticity would be 1.37 and 1.16×10^6 lbf/in^2 and the density would be about 23 lb/ft^3.

In our computer calculations we took some simple numbers which were intermediate to those just quoted, namely 8500 lbf/in^2 as the yielding strength, 1.2×10^6 lbf/in^2 as the Young modulus and 23 lb/ft^3 as the density. We then calculated the nuclear explosive yields which would just cause the poles to break; and these should be a close lower limit for Nagasaki and an upper limit for Hiroshima.

Let us first see what we can deduce without taking values of the properties of the wood. We construct scaling laws for poles in the blast wind, and then use the observation on the snapped pole at Nagasaki, 3300 ft from ground zero, to find a scaled pole at Hiroshima which would snap at a distance where drag conditions were most severe. We then compare the dimensions of the scaled pole with the dimensions of the poles which were observed not to have snapped.

The equation of motion of a pole of uniform cross-section in the blast wind is

$$EI\,\partial^4 y/\partial x^4 = 2rC_D\,q - m\,\partial^2 y/\partial t^2, \tag{13}$$

where y is the lateral displacement, x is the coordinate along the length of the pole, E is the Young modulus, I is the second moment of the cross-section about the neutral plane, r is the radius of the pole, q is the dynamic pressure at time t, C_D is the drag coefficient in the conditions at time t, and m is the mass of the pole per unit length.

We make the scaling laws

$$y_2 = ay_1, \quad x_2 = bx_1, \quad r_2 = \lambda r_1, \quad q_2 = \alpha q_1, \quad t_2 = nt_1. \tag{14}$$

Then
$$m_2 = \lambda^2 m_1, \quad I_2 = \lambda^4 I_1.$$

These substitutions imply that if y_1 satisfies the starting conditions y_2 also satisfies them.

The equation of motion scales if
$$b^2 = n\lambda, \quad a = n^2\alpha/\lambda,$$

and provided that C_D has not changed. (A small correction can be inserted if the average values of C_D for the two motions are not the same.)

If the whole system (height of burst, distance from ground zero, linear dimensions of pole) is scaled by the cube root of yield, the above equations are consistent, and therefore check as they should.

The bending moment at the base of the pole is obtained from the bending moment equation

$$M = EI\,\partial^2 y/\partial x^2.$$

The scaling law for the maximum stress in the cross-section is therefore

$$\sigma_2 = (n\alpha/\lambda)\,\sigma_1. \tag{15}$$

We shall now scale the pole which was observed to snap at Nagasaki to discover how it appears in Hiroshima conditions, and then compare it with poles there which did not snap.

The pole at Nagasaki was snapped by the blast wind behind the Mach stem. There was no peaking effect and the basic data are simple and reliable. For Hiroshima, we may choose any site where we think the conditions were most severe. Nearer to ground zero, we have to use a two-shock configuration: farther away, a single Mach stem but a peaked blast wave.

If, for trial purposes, we assume Hiroshima was 11 kT, then the greatest dynamic pressure in the two-shock configuration is 2.0 lbf/in^2, occurring between 2000 and 2200 ft from ground zero. Taking this to be the most severe site, we can use the scaling laws to find an analogue pole at this site of the Nagasaki pole which just snapped at 3300 ft from ground zero. We conclude that if the Nagasaki explosion had been 19 kT, the Hiroshima analogue pole would have had the required diameter but only a length of 23.2 ft compared with 25.5 ft of the actual Hiroshima poles; and the greatest stress at the base would have been only 76 % of the stress that caused failure. The extra length of the real pole would have put up the stress but not to 100 %.

Let us now look at the site at 2500 ft from ground zero, where the Mach stem is high enough to cover the poles, assuming that the buildings did not delay the formation of the Mach stem. With Nagasaki at 19 kT

$$q = 2.32, \quad \tau = 0.71,$$

and Hiroshima at 11 kT $\quad q = 2.40, \quad \tau = 0.63, \quad k = \tfrac{1}{3}.$

The scaling laws give $\quad \lambda = 0.85, \quad n = 0.67, \quad b = 0.75, \quad \alpha = 0.99.$

The length of the analogue pole is 20.3 ft and the stress is 78 % of the breaking stress. The natural period of the analogue pole was about 0.32 s, and half of this is longer than the duration of the 'peak' on the blast wave (the first Friedlander function lasts only 52 ms and the true blast is therefore more damaging than we have assumed). This pole would either have failed or would have been taken close to failure. So if Nagasaki was 19 kT, Hiroshima could not have been much more than 11 kT.

We have also calculated directly upper or lower limits to the yields of the explosions, using the observations on power line poles in the two cities. The equations of motion of the pole were solved on the computer by the same method as that used for the flagpoles in the elastic phase of the motion. Different values were taken form the nuclear explosive yield, and the yield which caused each pole to snap was obtained. The basic data on the wood have been given earlier. The bending moment to cause a cross-section of radius a to snap is $\frac{1}{4}\pi a^3 T$, where T is the tensile strength of the wood in bending, and we use for T a value 8500 lbf/in^2. Hence, the bending

moment to snap a pole of 8 in diameter is 4.3×10^5 lbf in. The drag coefficients we have used have been discussed in § 3, and are at high Reynolds number ($> 10^6$).

The snapped pole at 3300 ft from ground zero in Nagasaki would have been snapped by an explosion of 20 kT or more. The numbers for 20 kT are as follows. The natural period was 0.58 s. The peak dynamic pressure was 2.60 lbf/in^2 and the positive duration was 0.78 s. The drag coefficient began at 0.47.

The 17 ft pole which snapped at 2500 ft from ground zero in Nagasaki indicate a minimum explosive yield of 18 to 20 kT; and the poles at 1500 and 1800 ft indicate 15 to 18 kT.

On the assumption of the same mechanical constants of the wood as for Nagasaki, the fact that no power line poles like that shown in figure 14, plate 4, were snapped leads to the conclusion that a nuclear explosive yield of 14 kT would have caused poles at 2000 ft from ground zero to reach 90 % of their failure limit. At 2200 ft, assuming that the Mach stem covered the poles with peaking given by our basic data, 14 kT would have taken a pole just past its failure limit.

Summarizing then, the observations on the power line poles are of some value even though the mechanical properties of the wood are not known with certainty. They lead to the conclusion that if the Nagasaki explosion was not more than 20 kT, the Hiroshima explosion was less than 14 kT.

The bending of I *beam poles*

The interpretation of the bending of rolled steel joists used in Hiroshima as poles to carry wires crossing the street, to support the power line for the trams, logically belongs in the next section. We include it here because the basic ideas and the observations are closely similar to those relating to wooden power line poles already discussed.

Many different kinds of pole were used to carry the power lines; and along any part of a tram route, various types would follow each other without any obvious reason. Most of the poles made of steel were so complicated from the point of view of drag coefficient that the three Manhattan District observers made no attempt to record the damage, but the I beam poles were considered to offer some possibility of analysis.

The cross-section of the pole consisted of a web and two flanges. We call the angle of incidence θ of the blast the angle between the normal to the plane of the web and the radius vector from the pole to ground zero.

The nearest pole of this type to ground zero which showed any damage was north of the pole shown in figure 14 (three poles to the north). The set was about 5°, the distance from ground zero was about 1050 ft and the angle of incidence of the blast was 49°. The pole shown in figure 14 was set 12°, the distance from ground zero was 1300 ft and the angle of incidence was 36°. Proceeding farther south along the road, there were some I beam poles on the opposite side of the road. The set increased to about 20° and then near the Chugoku Electric Company Building one or two poles were prostrate (2200 ft from ground zero).

On the west side of ground zero, following the tramway route over the Shin Aioi Bashi bridge (the T bridge) there was a group of this type of I beam poles starting at 1750 ft from ground zero. One pole at 1750 ft was prostrate (angle of incidence 21°). The pole on the opposite side of the street (the south side) was very well shielded by a building and the pole was 'not bent much'. The next few poles farther away were bent 20 to 30°, and one of them was prostrate. The observers did not record whether there were any I beam poles beyond these and up to 2350 ft where the route turned north.

The cross-section of the beam which failed at 1750 ft was traced on to a sheet of paper. The height of the pole above ground level when vertical was within an inch or two of 26 ft. The outside dimension between the flanges was 6.0 in and the distance between the rounded tips of the flanges was 5.0 in. The tips were approximately semicircles of diameter $\frac{3}{8}$ in.

From the tracing, we have estimated the area of the cross-section and the first and second moments about the central line through the length of the cross-section of the web were 7.72 in^2, 6.75 in^3 and 9.90 in^4. Assuming that the yielding strength of the metal was 45 000 lbf/in^2, the yielding bending moment of the pole in the weaker direction would have been 304 000 lbf/in (765 000 in the direction at right angles). The period of the gravest mode in the weaker direction is estimated as 0.75 s.

We could make calculations on several of the observations described above but the one we choose is the pole shown in figure 14, 1300 ft from ground zero at an angle of incidence 36° and an angle of set of 0.21 rad. We find by interpolating between some of our computer solutions that the pole would have set 0.21 rad in a blast which gave a bending moment about its weakest axis of 850 000 lbf in and which had a positive duration 0.65 s (compared with the natural period of the pole 0.75 s). This gives a peak force normal to the plane of the web of 17.6 lbf per inch run.

Some information about the forces acting on an I beam in a transverse wind are available in the literature. Civil engineers usually quote the Swiss code and although these values were based on experimental wind tunnel data, the data are limited. However, some good measurements of the lift and drag at various angles θ for an I beam, $h = 6$ in and $b = 5$ in, have recently been made by Dr A. J. Taylor-Russell and Mr A. A. Mizra in connection with the forces on girders in a transverse wind. We are grateful for permission to quote some of their values before the complete data are published. Measurements were made of the drag (the force in the direction of the wind) and the lift (the force in the perpendicular direction) in terms of qh, for various angles of incidence θ. The sign convention on the direction of the lift was such that the force in the direction of the normal to the plane of the web was $(C_D \cos\theta + C_L \sin\theta)\, qh$. The lift coefficient changes sign at $\theta = 68°$. Over the range 30° to 45°, the drag coefficient is substantially constant but has a blunt maximum of 2.21 per unit length at $\theta = 40°$; and the lift is substantially constant at 0.34 per unit length. With $\theta = 36°$, the force resolved in the direction of the normal to the plane of the web is 1.96 qh. The error is thought not to exceed 5 %.

Since the Hiroshima I beam pole on which calculations were made had h as 6 in and the normal peak force was 17.6 lbf per inch run, the peak dynamic pressure was 1.50 lbf per in^2. If we take q from method 1 in table 1, the nuclear explosive yield was 10.9 kT; and if we take method 2, was 12.6 kT. The yield was therefore 12 kT with a reliability of the order 1 kT.

The other observations on bent I beams are all consistent with the above calculations but are less precisely defined.

5. Damage drag effects other than on circular cylinders

Memorial stones

Japanese cities have some or many small areas which are dedicated to the memory of the dead. Memorial stones are erected by families in honour of deceased relations.

There were many such memorial grounds in Hiroshima and Nagasaki. The area was sometimes rectangular and sometimes a less regular shape, with linear dimensions of several

tens of yards or more. The memorial stones were accurately cut and the exposed surfaces were highly polished with first-class craftsmanship.

The most common shape appeared to be a rectangular prism, of square or nearly square cross-section with a height between 2 and 4 times the length of a side of the cross-section. Sometimes the top of the stone was flat and square to the length, but more usually it was chamfered or slightly rounded over 2 or 3 in. Another common shape was a right triangular prism. There were a few cubes, usually large, with a side of 1.5 to 2.5 ft. There were several other shapes as well.

The effects of the blast were various. Some of the stones, especially cubes, had slipped on their highly polished plinths, and sometimes had then fallen off. Sometimes a stone had moved only a couple of inches and had rotated slightly. Many of the stones had toppled backwards and one or two seemed to have toppled forwards.

The stones which had toppled had usually been standing in a shallow recess in the plinth and had been rotated initially about the back edge of the recess. However, some stones of rectangular shape, moderately tall, stood freely on a highly polished plinth or pedestal and some of these had either slipped or toppled over, or possibly both. These were disregarded because tests indicated some doubt about whether they had slipped or rotated. Some of the stones still standing had obviously lifted and rocked because they were partially out of their recess. Some stones were still standing in the correct position in their recess although they must have rocked in the blast wave. This applied particularly to the rectangular prisms (X, Y, Z) with a smaller Z (height) than the toppled stones of similar (X, Y).

There were two possible types of measurement which could have been made. One was to look for stones (cubes in the main) which had just slipped on their plinth. The solution of the dynamics problems would have required a knowledge of the coefficient of friction. The Manhattan District observers quickly found that the coefficient of friction was extremely variable, no doubt due to a slight but variable atmospheric deposit on the plinth. The other type of measurement was to study which stones had toppled, there being no question of slipping because of the restraining effect of the edge of the recess in the plinth. The observers confined their measurements to toppled stones with the shape of rectangular prisms of square or nearly square cross-section. The objective was to discover the stones which had just toppled over in the positive phase of the blast wave. The first consideration was to decide whether the stone had been standing freely in its recess, or whether there had been some adhesion between the bottom of the stone and the recess. A number of stones were therefore examined carefully and tests were made on the spot.

The recesses in the stones appeared to have been chiselled out, the bottom left rough but the sides cut by a tool or saw. Sometimes the stone simply stood freely in its recess. Sometimes there was a slight circular shallow depression in the recess, covering about half of the area, centred on the middle. A little white setting material had been put in this depression, and the stone had been levelled on this material. The material might have been cement, but it looked more like plaster of paris and it was fragile. To obtain an indication of whether this material could hold the stone in place against a tilting force, several stones of widely varying heights and still in their proper positions were gently tilted. There was no indication of any restraint. It was therefore concluded that the stones were free standing. Even if there had been a little adhesion, the first few milliseconds of the blast acting on a stone which overturned would have parted the stone from the bottom of the recess, and the motion of the stone would have been unaffected. This argument is

based on the fact that a stone which was toppled by the blast was subjected initially to a much greater tilting force than that required to overcome the weight of the stone, especially during the first millisecond, before the flow pattern was established.

Some guiding principles had to be defined in order to select the stones which had only just overturned in the positive phase of the blast. The procedures adopted were as follows. Memorial grounds were usually selected where the blast was normal or nearly normal to one set of faces of the stones and where the distance from ground zero was sufficient to ensure that the Mach stem had formed. Otherwise the interpretation of the motion might have been more difficult. Stones with dimensions (X, Y, Z) with X equal, or nearly equal, to Y were sought. It was assumed that if the turning axis were taken as the Y axis, the stability of the stone against the blast for constant Z increased with increasing X; and for constant X, decreased with increasing Z. Thus, the three observers looked for square or nearly square section stones in a narrow range of X, and measured the dimensions of the shortest overturned stones. Each of the three observers measured a number of stones, choosing by eye and quickly checking overturned stones to get the required X value and with a Z value as small as possible. Stones most nearly critical could thus be chosen. Having found this stone, a search was made to see if a nearly similar stone still standing could be found, to check that a close limit had been obtained. The X values were measured to the nearest tenth of an inch. Because the stones were usually rounded off at the top, the average Z dimension as judge by eye was measured to the nearest half an inch. At the time, the Y dimension was considered to be immaterial, but the Y values were recorded.

Figure 15, plate 5, shows a photograph of some of the stones in a memorial ground at Hiroshima. In this memorial ground, square section stones with $X = Y = 9.8$ in were very common. There were several tens of such stones, most with Z/X between 2.5 and 3.5. The stone which was considered critical was $9.8 \text{ in} \times 9.8 \text{ in} \times 28.5$ in. Five examples of overturned stones with these dimensions were found but none with a smaller Z dimension was overturned. The picture shows among others a few of the stones with $X = 9.8$ in, and with heights in the range 25 to 35 in. The picture appears to have just caught a stone which had toppled forwards. The stone from the first plinth in the second row from the bottom left-hand corner appears to have tumbled towards the blast, which came from the left-hand side of the picture. The front edge of the plinth appears to be chamfered.

Another common stone in the Hiroshima memorial ground had a square cross-section with $X = Y = 6.5$ in. Many such stones, with Z values about 18 in, were found. All were overturned but one was recorded as having overturned forwards, towards the blast. (We cannot account for this particular observation.)

The stones in a memorial ground at Nagasaki were on the average larger than those observed in a memorial ground at Hiroshima. The most common stone of square or nearly square cross-section had a side about 14 in. The stone which was considered to be most nearly critical, and which had overturned, was rectangular but nearly a square.

The stones which were measured as most near to critical were made from a material with bold black and white markings (presumably granite). A few pieces of broken stones from both cities were taken to England, where the density was measured to be 2.57 g/cm^3. There were also many stones in Nagasaki made from another material. (The density was 2.38 g/cm^3.)

The observations were as follows.

Hiroshima

The memorial ground was adjacent to the Kokyo Temple and was 4280 ft east of ground zero, just to the north of the main east–west road where the trams operated. The dimensions of the memorial ground were 150 ft east to west and 70 ft north to south. At the time of the explosion, the area around the memorial ground was densely built-up with one- or two-storey Japanese houses. Five identical stones, considered to be very close to the condition of having just overturned, had the dimensions $X = 9.8$ in, $Y = 9.8$ in, $Z = 28.5$ in. The angle of incidence of the blast wave was $10 \pm 2°$. These stones were all of the highly polished mottled black and white granite, density 2.57. The Manhattan District observers were in no doubt that these stones had overturned and had not slid off their plinths.

Nagasaki (*Nishigo*)

This memorial ground was north of ground zero, on the west side of the railway line, with the Torpedo Factory on the other side.

The distance from ground zero was 4610 ft and the angle of incidence of the blast to the stones was 22°. There were a number of stones of square or nearly square cross-section with side about 14 in and the overturned stone which was considered to be very near to critical had the dimensions $X = 14$ in, $Y = 16$ in, $Z = 38$ in. This stone was also highly polished mottled black and white granite with density 2.57. The observers were satisfied that this stone had overturned and not slipped.

The following argument, which makes several approximations, is useful for purposes of quick orientation. Suppose that a stone (X, Y, Z) of density ρ is given impulsively an angular momentum representing the effects of a blast wind at normal incidence. The impulsive angular velocity is proportional to $q_0 \tau/\rho XZ$, where q_0 is the peak dynamic pressure and τ is the positive duration. This angular velocity is to be just sufficient to topple the stone, a condition which requires the impulsive kinetic energy to be $\rho g X^3 Y/4$. Hence $q_0^2 \tau^2$ is proportional to $\rho g X^4/Z$ so that at any one site, the condition that characterizes the class of stones which just overturned is X^4/Z is constant. The stability of stones of constant Z increases rapidly with increasing X, but the stability of stones of constant X decreases rather slowly with increasing Z.

The equation of motion of a stone with sides X, Y, Z about the back bottom Y edge is

$$I\ddot{\theta} = C_y(t) - \tfrac{1}{2}Mg(X\cos\theta - Z\sin\theta), \tag{16}$$

where I is the moment of inertia about the turning edge, $C_y(t)$ is the turning moment due to aerodynamic forces about the turning edge at time t, M is the mass of the stone and θ is the (small) angle of turn.

The starting conditions are

$$t = 0, \quad \dot{\theta} = 0, \quad \theta = 0. \tag{17}$$

The condition which implicitly determines the yield of explosion which would just overturn the stone is that when $\dot{\theta}$ returns to zero the centre of gravity of the stone is above the turning edge, i.e.

$$\theta = \tan^{-1}(X/Z), \quad \dot{\theta} = 0. \tag{18}$$

A simple but imprecise estimate of the function C_y may be obtained in the following way. Most of C_y comes from the moment of the aerodynamic forces over the four largest, and initially vertical, faces of the stone. At normal incidence, the resultant force over the faces is the drag, and its line of action is about half way up the stone. So in units of the dynamic pressure q, C_y is

approximately $\frac{1}{2}C_D YZ^2$. We cannot put a value on that part of the overturning moment coming from the aerodynamic pressure (actually a suction) over the top of the stone. However, as soon as the stone tilts, the aerodynamic pressure over the base must be slightly less than the dynamic pressure, and its moment a little less than $\frac{1}{2}qX^2Y$.

The values of the drag coefficient for rectangular prisms of square cross-section have been measured by several observers, and the results can be summarized as follows. For prisms very long compared with the cross-sectional dimensions, the drag coefficient is nearly 2.0; while for cubes, the drag coefficient is slightly greater than 1.0. For prisms with Z/X about 3, the drag coefficient is slightly less than 1.5. In addition to these published data coming from wind tunnels we also have a recent value obtained in a shock tube.

J. E. Uppard, in a private communication to us (1968), has measured as 1.38 the drag coefficient of a square prism with a Z/X ratio 2.8, standing on a plate in an air shock tube with a flat top shock pulse of overpressure 5.8 lbf/in². A point of importance is that the drag coefficient of a rectangular prism is independent of Reynolds and Mach numbers with close approximation, and certainly within the possible experimental error in existing measurements.

The best value for C_y would appear to be obtainable from some measurements by Dryden & Hill (1926). Measurements of the aerodynamic pressure were made at about 350 points on the four faces and top of a right square prism 8 in × 8 in × $24\frac{1}{2}$ in standing on the floor or on a platform to represent the ground, in a wind tunnel. Among the data given are the moments of the pressures, expressed in terms of the dynamic pressure q, on the front, side, and back (upright) faces, about an axis through the centre of the base parallel to the bottom back edge. Basic data are also given for the moment of the aerodynamic pressure over the top, about the same axis. These data are given for various angles of incidence of the wind, 0°, 15°, 30°, 45°. Over most of the top of the prism the aerodynamic 'pressure' is nearly constant and equal to $-0.8q$. Over the base of a tilted stone, the aerodynamic pressure must be very nearly constant at a value slightly less than q.

We may therefore say that the moment of the aerodynamic forces on the top of the prism, about the bottom back edge, is the same as that of the aerodynamic forces over the base. Averaging the results of Dryden & Hill for the prism on the floor and on the platform representing the ground, and doubling the moment for the top to take account of the moment of the forces over the (tilted) base, we find that for an angle of incidence ϕ the data can be accurately fitted by

$$C_y/q = (0.774 - 0.159 \sin 2\phi)\, YZ^2 + 0.85YX^2.$$

Irminger & Nøkkentved (1930) have made similar measurements on a right square prism of dimensions 44 mm × 44 mm × 99 mm but did not have as many pressure points as Dryden & Hill. Their results are in good agreement with those of Dryden & Hill but the coefficient of YZ^2 is about 10 % less and the coefficient of YX^2 is about 15 % less. The formula which we think best is obtained by fitting the drag at normal incidence to the coefficient 1.38 measured in the shock tube and by using the centroids as observed by Dryden & Hill and by Irminger & Nøkkentved. The variation with angle of incidence is best determined by the results of Dryden & Hill. Our formula is

$$C_y/q = (0.677 - 0.139 \sin 2\phi)\, YZ^2 + 0.72YX^2, \quad \phi < 30°. \tag{19}$$

Hiroshima

We have solved equation (16) with the overturning moment given by equation (19) to find what value of the nuclear explosive yield at Hiroshima would have just overturned the stones in the memorial ground of the Kokyo Temple, described earlier. The nuclear yield that fits the

FIGURE 18. The ladder called the 'first ladder' in the text.

FIGURE 19. The ladder called the 'third ladder' in the text.

FIGURE 20. A piece from the top of the wall around the prisoners of war camp in Nagasaki thrown by the blast.

(Facing p. 388)

FIGURE 23. The building in Nagasaki where the tool cabinets were found.

FIGURE 24. The Chugoku Coal Distribution Company at Hiroshima where the safe was found.

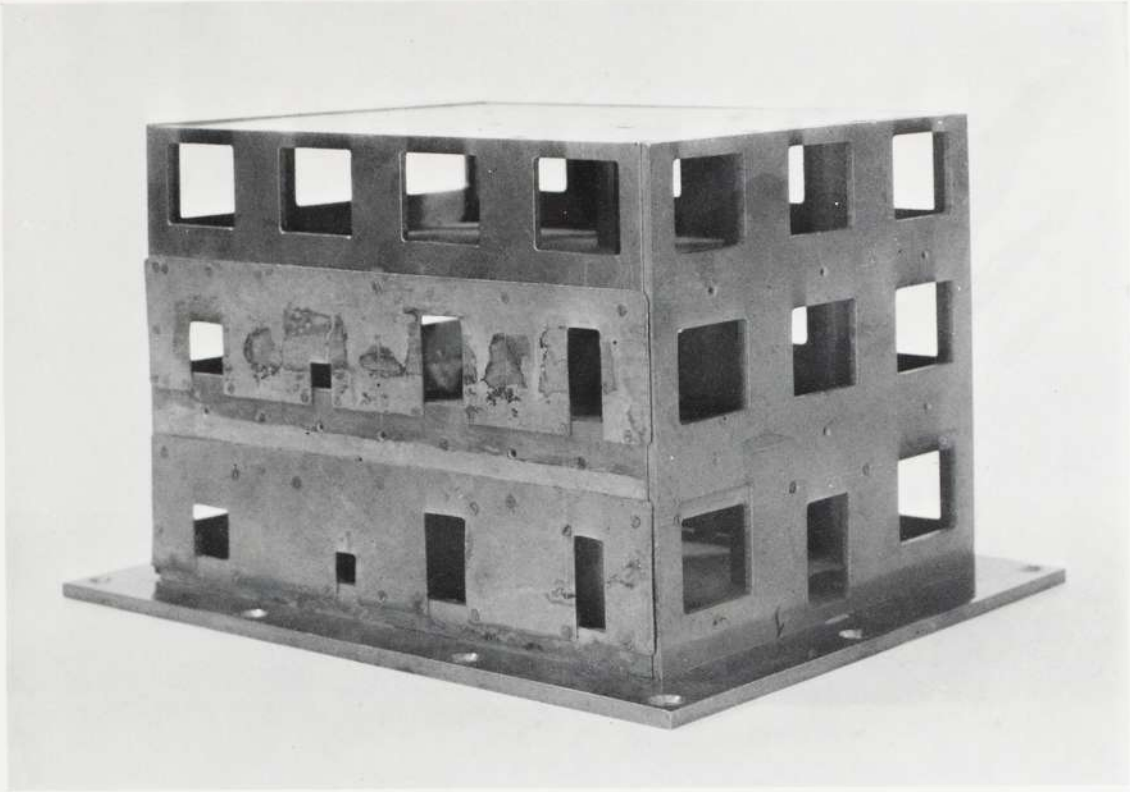


FIGURE 26 Model of Chugoku Coal Distribution Company building.

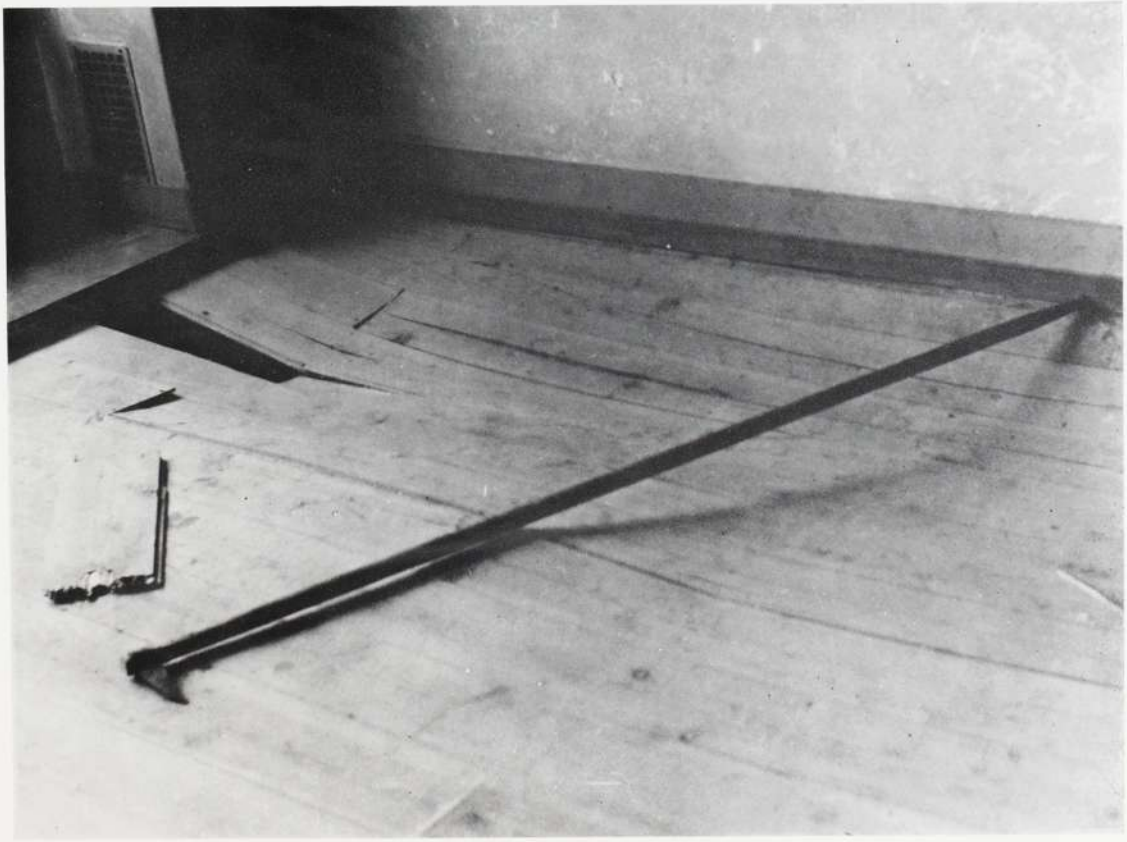


FIGURE 29. The panel which failed of the platform in the Telephone Exchange, Hiroshima. The damage at the left-hand end was made during the examination of the panel. The dish under the wooden rod is blast damage.

FIGURE 31 (above). A 46-gal empty 16 B.S.W. drum with central filler cap missing collapsed by the blast at Hiroshima.

FIGURE 33 (right). The blue-print container at Hiroshima.

FIGURE 32. A thin wall empty can with filler cap missing, collapsed by the blast at Nagasaki. The 46-gal drum was full of water.

observations is so low that according to our basic data there would have been no peaking in the blast wave.

A peak dynamic pressure of 0.60 lbf/in^2 with a positive duration of 0.64 s would just have overturned stones 9.8 in × 9.8 in × 28.5 in. This corresponds with a nuclear explosive yield of $7\frac{1}{4}$ kT, exploded over bare ground.

The three Manhattan District observers thought that they had identified stones which had only just overturned. Stones with the same cross-section, but slightly shorter, had not overturned. They only recorded 'very close' and did not write down the height of the shortest stones with this cross-section which did not overturn. However, figure 15, plate 5, shows a stone of the same cross-section which did not overturn. The height was three quarters of that of the stones which did overturn. If our analysis is correct, a nuclear explosion of 9 kT, exploded over bare ground, would have just overturned this shorter stone. The nuclear yield must therefore have been less than this.

We will consider later the corrections we must apply to allow for the shielding given to the stones by the houses which surrounded the memorial ground.

Nagasaki

Similarly, we have calculated the explosive yield at Nagasaki which would account for the observations on the stone which just overturned in the memorial ground at Nishigo. The nuclear explosive yield so obtained was 20 kT.

Hiroshima and Nagasaki

We have found some additional data on memorial stones which were overturned in the two cities by the explosions. A team of British observers from the Ministry of Home Security studied the blast damage at Hiroshima and Nagasaki in late 1945, and among the observations they made were some on overturned memorial stones. They measured the largest and most impressive stones which were overturned at several memorial grounds. We have calculated the lowest explosive yields which would have overturned the stones which were recorded as having overturned, but in most cases the computed lower limit to the yield is too low to be of much value. Thus the observers recorded several stones at Hiroshima at normal incidence 1650 ft from ground zero which had overturned, and the stone which gave the greatest lower limit to the yield had the dimensions 11.5 in × 10 in × 23 in. A yield of 8 kT would just have overturned this stone. At a distance of 3750 ft from ground zero, at nearly normal incidence, the most stable stone which was recorded as having overturned was 9.5 in × 9.5 in × 22 in but this gives a lower limit (only $5\frac{1}{2}$ kT).

The most interesting observation made on overturned stones by the British Ministry of Home Security observers was 5430 ft a little west of due north from ground zero at Nagasaki. The dimensions of the stone were 12 in × 13 in × 28 in and the angle of incidence was 23°. A photograph of this overturned stone is shown in figure 16, plate 5. We find that a yield of 21 kT would have just overturned the stone, assuming the stone to be granite (density 2.57). However, the stone was probably not granite and it may have been made of another material commonly found in the memorial grounds and whose density was measured at 2.38. In this case, the nuclear yield is calculated as greater than $19\frac{1}{2}$ kT.

We have checked that the aerodynamic forces would not have caused any of the stones also to turn about one of the X axes at the base.

The estimates of the nuclear yield at Nagasaki agree well with our other estimates and with the expected value; but the Hiroshima yield is low. We shall discuss the possible explanations.

The Nagasaki memorial ground was at an elevation of about 10 m above the level ground zero. The memorial ground itself was level but the ground was rising towards the north-west. We need an order of magnitude calculation to estimate the effects of the rising ground on the blast wave and hence to determine the approximate error in the estimated yield.

Friedlander (1946) and Lighthill (1949), by different methods, have discussed the diffraction and reflexion of pulses. Friedlander's solutions are more easily adapted to our case. Although the treatment is effectively sound theory, the pulses are similar in shape to those of a blast wave. Among the diffraction problems solved by Friedlander is that of a plane pulse led by a pressure step and decaying exponentially, incident symmetrically on the apex of a wedge. The 'length' of the pulse is taken to be the distance over which the overpressure falls to $1/e$ of its value at the front. If the (internal) angle of the wedge is 2θ, then the pressure at the apex varies with time in exactly the same way as the pressure in the pulse before it reaches the apex, but the pressure is enhanced by the factor $\pi/(\pi-\theta)$. The 'positive impulse', i.e. the time integral of the overpressure, at the apex is increased in the same ratio. The positive impulse at a point on the surface of a wedge of small angle at a distance equal to two or three times the pulse length is practically the same as it is at the apex; although at many times the pulse length the positive impulse increases, until eventually it is doubled, corresponding with the normal laws of reflexions at a rigid plane of infinite extent.

In our application, we imagine the system divided into two equal parts by the central plane of symmetry. We then have a plane pulse travelling parallel to the ground and meeting a hill which thereafter continues indefinitely. The 'length' of the pulse in our case is about 500 ft. To obtain an idea of the possible effect of the non-horizontal ground around the memorial ground at Nagasaki we suppose that the memorial stone was on a hill rising in the direction of travel of the blast. The stone is taken to be 40 ft above the foot of the hill which is 500 ft away, and the hill continues to rise by a further 40 ft in the following 500 ft. This model probably exaggerates the effect of the non-level ground near the stone.

The angle θ is 0.08 rad, and the positive impulse at the site of the stone is enhanced by 2.5 %. The time integral of the dynamic pressure will be increased by 5 %. The nuclear yield will be found to have been overestimated by 2.5 %, and the estimate must therefore be reduced from 20 to 19.5 kT.

The blast wind acting on the stones in the memorial ground of the Kokyo Temple at Hiroshima was reduced by the presence of the houses surrounding the ground. We can make an assessment of this effect from the results of two different experiments. Worsfold (1957) made some model field experiments with explosive charges to compare the blast loading on a house when the house was standing alone on flat ground and when there was some shielding given by another house nearer to ground zero. Scaling the length of the pulse to that in Hiroshima, the houses were 22 ft high and also 22 ft in the direction of travel of the blast. The other dimension, tangential to the blast front, was 27 ft (single houses) in one set of tests; and 1000 ft (terraces) in another set. Several spacings were taken, up to 100 ft. From a study of these records we have reached the following conclusions.

If the memorial ground and the surrounding houses had been sited on a completely flat area free of all other structures, the blast wind at the central region of the memorial ground would have been the same as it would have been over open flat ground from the time of arrival of the

blast until a time $0.2t_+$. A reflected pulse would then have been superimposed, coming from a reflexion on the walls of the houses at the far end of the memorial ground. This reflected pulse would have lasted until $0.28t_+$, and at ground level at the centre of the memorial ground its peak amplitude would have been only 20 % of that of the peak of the original pulse. The reflexion of the reflected pulse, coming from the near end of the memorial ground, would have arrived at about $0.4t_+$, but it would have been so small that its effect on the dynamic impulse would have been completely negligible.

The time integral of the dynamic pressure (the dynamic impulse) acting on the memorial stones was therefore reduced by only 3 %.

Mr K. F. Mead has made some model experiments in a shock tube to estimate the effect of the houses surrounding the memorial ground. The models were solid and immovable. The dynamic pressure was measured at the middle of the memorial ground at ground level, using a total head gauge, and compared with the value in the absence of the model. The cross-section of the shock tube was 30 in × 18 in. Two tests with different step pulses were made.

The reflected pulse was of short duration and was not properly resolved. The duration is approximately equal to 1.5 times the time taken by sound to travel the height of the houses at the far end of the memorial ground. The weakening of the direct wave at ground level at the centre of the memorial ground by diffraction at the upstream houses is more important than the reflected pulse.

The results of the two tests were:

measured side-on overpressure free stream (lbf/in^2)	8.1	6.8
calculated free stream stagnation pressure	9.7	7.9
calculated free stream dynamic pressure, q	1.45	1.05
measured stagnation overpressure (shielded)	7.55	6.55
calculated dynamic pressure (shielded)	1.28	0.98
percentage reduction in q	12	7

The dynamic impulse acting on the stones near the Kokyo Temple was therefore reduced by about 10 % by the shielding effect of the houses around the memorial ground. The nuclear explosive yield is calculated as 8 kT. This is low compared with the values indicated by observations nearer to ground zero. The probable explanation is that the blast near the ground was causing so much mechanical damage, and being scattered by buildings, that at a distance of 4280 ft from ground zero, the blast wind was less than it would have been with the same explosion over bare ground.

There must have been some diminution of the blast at Nagasaki by the time that the blast had reached the memorial stones on which calculations have been made. However, this diminution was probably slight. The sites where the stones were overturned at Nagasaki were open, and the radius vectors to ground zero did not pass over such heavily built-up areas as was the case in Hiroshima. Moreover, the slightly rising ground at Nagasaki would, as it were, have chopped off the weakened part of the blast near the ground.

Our conclusion is that the memorial stones indicate 20 to 22 kT at Nagasaki; but at Hiroshima a value (8 kT) which is low because of mechanical damage and scattering.

Bending of mild steel roof ladders

Three similar mild steel ladders on the roof of the City Offices in Hiroshima were bent sideways at the top by the drag effects of the blast. The building was a large one with an auditorium at the west side, and the centre of the building was some 3300 ft from ground zero. Figure 17, plate 4, shows a view of this building.

The roof of the City Offices was not all at the same level and the ladders were provided to give access from one part to another. The ladders went up one side of walls 12 to 15 ft high, over a parapet wall and down on the other side of the parapet wall which was some 2 ft 6 in in height Iron fish-plate platforms were rivetted to two pieces of angle iron which were also rivited to the two arms on each side of the ladder. Thus, the angle irons were at right angles to the ladder steps. The platforms cleared the parapet wall by a few inches.

Calling the ladders the first ladder, the second ladder and the third ladder respectively, the first and second were of identical construction and dimensions but the second ladder was partially shielded from the blast. The third ladder differed in that it spanned a wider wall. The ladders were all 10 to 20 ft inside the southern edge of the roof of the building and their distance from ground zero was 3320 ft. The angle of incidence of the blast to the plane containing two arms of the same side of a ladder was 15° in each case. Figures 18 and 19, plate 6, show photographs of the first and third ladders.

The ladders were attractive in appearance and looked as if they had been custom made in some small local shop. They were made of a mild steel strip which had been heated to shape the top arms; and the width of the metal at the curve was greater than at the straight parts (and was about 2.3 in) owing to the shaping and hammering of the curved part. Although no sample was taken to England for analysis, the yield strength can be taken as 35 000 to 40 000 lbf/in^2. We use the higher figure since we are particularly interested in getting a value to the yield which is more likely to be on the high side than the low.

The first ladder was severely bent at the top rungs on the two sides of the wall, at the points of support to the wall. Above this bend the arms of the ladder had remained straight. (Three of the arms are certainly straight: the front, right-hand arm appears to be very slightly bent.) The third ladder was severely bent at the platform and angle iron level, and also at the level of the top rung. There was a slight bend between the top rung and the next one. The bend around the angle iron was a double bend, forwards just above the platform and backwards just below. With the first ladder, the strength of the attachments of the platform and angle irons to the arms had caused the corners of the platform to bend, rather than the arms just below the angle iron bending retrovertedly, as in the third ladder. Idealized, in the third ladder, the arms were clamped vertically over a small distance where the angle irons were fixed. Above the clamp, the arms were bent in the direction of the blast wind: below the clamp, the arms were bent in the opposite direction.

The details were as follows:

First ladder

cross-section of metal arms (in)	1.94×0.325
distance across ladder to outsides of metal arms (in)	17.8
height of ladder step (in)	11.2
distance from mid-point of top rung to top side of top cross-arms (in)	37.4

distance from mid-point of top rung to top side of platform (in)	11.4
diameter of rung (in)	0.65
angle iron supporting platform (in)	$1.5 \times 1.5 \times \frac{3}{16}$
platform dimensions (in)	$17.2 \times 19.5 \times \frac{3}{16}$
outside radius of quarter circles turning top arms into vertical (in)	4.0
angle of set of first ladder	$20° = 0.35$ rad

Second ladder

identical with first ladder but angle of set	$14° = 0.24$ rad

Third ladder

Same as above except for the following variations:

platform dimensions (in)	$17.2 \times 23.8 \times \frac{3}{16}$
angle of set at upper bend	$20° = 0.35$ rad
angle to the vertical at lower bend	$12° = 0.21$ rad
angle of upper part of arm to vertical	$32° = 0.56$ rad

A detailed point of difference between the first and third ladders is mentioned here and the effects discussed later. On the short side of the wall, both ladders were fixed to the wall at the top and bottom rungs, and the rungs were clamped to the ladder arms by pairs of nuts rotating on screw threads on the ends of the rungs where the rungs passed through a hole in the arms. The first ladder was similarly fixed to the walls at the top rung on the tall side of the wall, and at every fourth rung below the top one. The third ladder was similar, except that the top rung had its screw threads and nuts, but the fixing to the wall at the top rung had been omitted. All other rungs on both ladders penetrated the ladder arms at their two ends, and the small projections were burred over tight by hammering (and rust).

The dynamics of the elastic bending of the ladders in the blast is extremely complicated and we have not attempted to make full computer calculations. However, the angles of set were large enough to justify ignoring the elastic flexures, taking account only of inertia terms and plastic yield. The geometry of the shock wave system at the site of the City Office must have been similar to that at the Hypothec Bank, 3080 ft from ground zero. The Mach stem would have reached nearly to the top of the building but would not have covered the ladders. The blast wind would have been enhanced by 10 %.

Let us start with calculations on the first and second ladders and obtain an expression for the total bending moment acting on the four arms at the point of support to the wall (i.e. the midpoint of the top rung). We assume that the yield strength of the material was 40 000 lbf/in^2. We ignore the weakening effect of the 0.65 in holes through the arms through which the rung passed, to be held by nuts on a screw thread and by the arms fixed in the wall. (This assumption has been shown to be justified by making a full scale model of the fixings. The loss of strength was 2 % with the nuts fixed tightly.)

The resistance to the sideways bending of the ladder at the point of support to the wall came from the resistance to bending of the four arms plus the resistance of the fixings holding the platform and angle irons to the arms of the ladder. The yielding bending moment of the system of the four arms is

$$M_Y = 4 \times 1.94(0.325)^2 \times 40\,000/4$$
$$= 8200 \text{ lbf in.}$$

Our full-scale model experiment using black iron bar (British Standard Specification B.S. 15) gave the yielding bending moment per arm as 2040 lbf in, in good agreement with the bending moment we have deduced from the assumed yield strength.

We do not know the resistance to sideways bending given by the rivets and platform and angle irons, but it could not exceed M_Y because, if it had, the arms of the ladder would have had to bend in two places—one at the point of support to the wall and the other, in the reverse direction, just below the points were the angle irons were attached to the arm (which was the situation with the third ladder, together with a bend just above the platform).

Therefore, the couple resisting bending was certainly appreciably greater than M_Y and was less than $2M_Y$. We shall calculate lower and upper limits to the yield of the explosion by taking the resisting couple first as M_Y and second as $2M_Y$.

Let M_R be the couple resisting bending. Let S be the drag pressure at time t. Then the bending moment at the points of support to the wall, where bending occurred, was

$$M_{\mathrm{D}} = 7785S \,\mathrm{lbf\ in}.$$

The equation of motion is $$\ddot{\theta} = g(M_{\mathrm{D}} - M_R)/I,$$

where θ is the angle of the arms to the vertical and I is the moment of inertia, about the plane of the bends, of the arms, the angle irons and the platform.

In this equation of motion, to keep the same dimensions for each term, we express g in inches per second squared, so that g is 386 in s^{-2}.

To obtain the drag pressure, one needs the drag coefficient C_{D}. The dynamic pressure $q(t)$ must be increased by 20 % to allow for the enhancement of the blast wind on the ladders caused by the building.

$$\begin{aligned} S &= 1.20q(t)\,C_{\mathrm{D}} \\ &= S_0(1-4T+7T^2-22T^3/3\ldots), \end{aligned}$$

where $T = t/\tau$, and τ is the positive duration of the positive impulse.

The equation of motion becomes

$$\ddot{\theta} = (g/I)\,[M_{\mathrm{D}}^0(1-4T+7T^2+\ldots)-M_R], \tag{20}$$

where M_{D}^0 is the peak value of the bending moment caused by the drag pressure at time zero.

Write $$\mathrm{d}^2\theta/\mathrm{d}T^2 = A_0 + A_1\,T + A_2\,T^2\ldots,$$

where $$\begin{aligned} A_0 &= (g\tau^2/I)\,(M_{\mathrm{D}}^0 - M_R), \\ A_1 &= -4(g\tau^2/I)\,M_{\mathrm{D}}^0, \quad \text{etc.} \end{aligned}$$

The boundary conditions are

$$\theta = 0, \quad \dot{\theta} = 0 \quad \text{at} \quad T = 0.$$

Provided that yield is occurring, i.e. provided $\dot{\theta} > 0$

$$\theta = \tfrac{1}{2}A_0\,T^2 + \tfrac{1}{6}A_1\,T^3 + \ldots.$$

The motion stops when $\dot{\theta}$ again reaches zero at time T_1. The final angle of set is therefore

$$\theta_\infty = \tfrac{1}{2}A_0\,T_1^2 + \tfrac{1}{6}A_1\,T_1^3 + \ldots. \tag{21}$$

The observed angle of set was 20° or 0.35 rad. The solution can be found by successive approximations.

The drag coefficient of a thin long rectangular plate (length to width ratio of 10 or more) is known with good accuracy from the work of several observers to be between 1.55 and 1.8 (see, for example, Irminger & Nøkkentved 1930). The dependence on Reynolds and Mach numbers is slight and can be ignored. Causing the long axis to be inclined up to 30° to the wind direction makes no difference to the normal force, but inclining the short axis by 15° reduces the normal force by about 5 %. The presence of a second arm at a distance some ten times the width of the first one and located behind it, particularly for incidence which is not quite normal, gives a drag on each arm very nearly as great as if the other were not there. We therefore consider that a value of 1.6 for C_D is correct within 10 %, and this assumed value is not likely to be on the low side.

TABLE 7

yield/kT ...	8	9	10	11
q/lbf in^{-2}	1.158	1.287	1.444	1.670
τ/s	0.62	0.65	0.67	0.69

To obtain the numerical solution of the equation of motion of the ladders, we need the peak dynamic pressure and positive duration of the dynamic pressure at 3300 ft from explosions of various yields, burst at a height of 1890 ft. The basic data are given in table 7. Calculating the numbers, it will be found that for the first ladder the moment of inertia is 20 900 lb in^2.

The lower limit for the yield of the explosion, corresponding with the resistance of the first ladder to bending coming only from the resistance to bending of the four arms, is only slightly greater than 5 kT, but this is not realistic. The upper limit where the resistance to bending is that required to bend the four arms at two places, is 11.0 kT and the non-dimensional time T_1 is 0.22, which in real time is 150 ms. The evidence of the third ladder showed that the angle iron and platform fixings were nearly as strong as the arms. Thus the explosion yield must have been about 11 kT.

The observations on the second ladder lead essentially to the same conclusion. Although the angle of set of the second ladder was only 0.24 rad, compared with 0.35 rad for the first ladder, the ladder was partially shielded from the blast.

We turn now to the interpretation of the observations on the third ladder. This time we have to fit the two angles of set with a single disposable parameter.

The idealized model to represent the ladder is as follows. A rigid rod of length l_1 and mass m_1 (representing the four arms of the ladder between the two bends) is attached by a plastic hinge to a support, while at the other end is a mass M_3 (representing the mass of the platform and of the angle iron supports) and another plastic hinge. To the second hinge is attached a rigid rod of length l_2 and mass m_2 (representing the originally vertical parts of the four arms of the ladder above the platform) and a mass M_4 at the free end (representing the two horizontal parts of the top of the ladder). The upper plastic hinge does not bend if a couple less than magnitude C_2 is applied, while if it is bending the resistance is a couple C_2. Similarly, for the lower plastic hinge, when bending is occurring the resistance couple is C_1. (Couple C_2 represents the bending moment of the four arms: couple C_1 is twice as great but we take a general value in the analysis to see how the theory develops.) There are four external horizontal forces applied to the system. A force F_1 acts at the mid-point of the first rod (representing the drag on the four arms of the ladder between the two bends); a force F_2 acts at the mid-point of the second rod (representing the drag on the vertical parts of the four arms above the upper bend); and a force F_4 acts on M_4 (representing the drag on the two horizontal top parts of the ladder) and a force F_3 acts on the mass M_3 (representing the drag on the angle iron and platform).

The angle of inclination of the lower rod to the vertical at time t is θ_1; and of the upper rod is θ_2. Let $\pm R$ be the (horizontal) reaction at the top of the lower rod at time t.

The horizontal velocity of the top of the lower rod is $l_1\dot{\theta}_1$; and of the top of the upper rod is $l_1\dot{\theta}_1+l_2\dot{\theta}_2$.

The effect of gravity is so small that it may be neglected, and we also neglect the effects of the slight bend at the second rung from the top.

The equation of linear momentum of the upper rod gives

$$M_4(l_1\ddot{\theta}_1+l_2\ddot{\theta}_2)+m_2(l_1\ddot{\theta}_1+\tfrac{1}{2}l_2\ddot{\theta}_2) = F_2+F_4-R.$$

The equation of angular momentum of the lower rod gives

$$M_3 l_1^2\ddot{\theta}_1+\tfrac{1}{3}m_1 l_1^2\ddot{\theta}_3 = Rl_1+\tfrac{1}{2}F_1 l_1+C_2-C_1.$$

The equation of total angular momentum of the system gives

$$\begin{aligned}&\tfrac{1}{2}F_1 l_1+F_3 l_1+F_2(l_1+\tfrac{1}{2}l_2)+F_4(l_1+l_2)-C_1\\&\quad=\tfrac{1}{3}m_1 l_1^2\ddot{\theta}_1+M_3 l_1^2\ddot{\theta}_1+M_4(l_1+l_2)(l_1\ddot{\theta}_1+l_2\ddot{\theta}_2)+(m_2/l_2)\int_0^{l_2}(x+l_1)(l_1\ddot{\theta}_1+x\ddot{\theta}_2)\,\mathrm{d}x.\end{aligned} \tag{22}$$

The reaction R can be eliminated between the first two equations. The eliminant, together with the third equation, make a pair of simultaneous equations for $\ddot{\theta}_1$ and $\ddot{\theta}_2$. The symbolism used above can be in any set of absolute units, but in practice for the arithmetical calculation we shall use pounds weight for force and inches for length. As with the first ladder, we have to insert a 'g' expressed in inches per second squared to keep the units consistent. Then the solution of the equations gives:

$$\left.\begin{aligned}\ddot{\theta}_1 &= [g/(\alpha\delta-\beta\gamma)]\,[l_1(\delta-\beta)\,(\tfrac{1}{2}F_1+F_2+F_3+F_4)-l_2\beta(\tfrac{1}{2}F_2+F_4)\\&\qquad\qquad+\delta C_2+(\beta-\delta)\,C_1],\\ \ddot{\theta}_2 &= [g/(\alpha\delta-\beta\gamma)]\,[l_1(\alpha-\gamma)\,(\tfrac{1}{2}F_1+F_2+F_3+F_4)\\&\qquad\qquad+l_2'\alpha(\tfrac{1}{2}F_2+F_4)-\gamma C_2+(\gamma-\alpha)\,C_1],\\ \alpha &= l_1^2(\tfrac{1}{3}m_1+m_2+M_3+M_4),\\ \beta &= l_1 l_2(\tfrac{1}{2}m_2+M_4),\\ \gamma &= \alpha+\beta,\\ \delta &= \beta+l_2^2(\tfrac{1}{3}m_2+M_4).\end{aligned}\right\} \tag{23}$$

The boundary conditions are

$$t=0,\quad \theta_1=\theta_2=0,\quad \dot{\theta}_1=\dot{\theta}_2=0. \tag{24}$$

Bending motion at both plastic hinges continues until the lower one sets, the time value T_1 being determined by the equation

$$\dot{\theta}_1=0,\quad t=T_1. \tag{25}$$

At time T_1 the motion continues as a rigid system about the upper plastic hinge. The equation of motion is

$$\ddot{\theta}= [g/l_2^2(M_4+\tfrac{1}{3}m_2)]\,l_2(\tfrac{1}{2}F_2+F_4), \tag{26}$$

with the boundary conditions $\qquad t=T_1,\quad \theta=\theta_2,\quad \dot{\theta}=\dot{\theta}_2.$ $\qquad$ (27)

This motion continues until time $t=T_2$, given by the equation

$$\dot{\theta}=0. \tag{28}$$

The motion then stops, and the value of θ at this time is the final angle between the upper part of the arms and the vertical.

We now try to fit the observations $\theta_1 = 0.21$, $\theta_2 = 0.56$ with a single choice for the yield of the explosion, and with $C_1 = 2C_2$, $C_2 = 8200$ lbf in, the yielding bending moment of the four arms.

A nuclear explosive yield of 10 kT fits both angles of set within 20 %. The lower bend sets at 100 ms and the upper bend sets at 150 ms.

We could get an exact fit to θ_1 and θ_2 by choosing both the yield and the ratio of C_1/C_2. To do this, we have to go to the first decimal figure in the yield and make C_1/C_2 slightly greater than 2. However, we do not consider that the extra numerical accuracy has any significance. The two angles are sensitive to the yield of the explosion, and a few per cent change in the ratio of C_1 to C_2 throws one angle rapidly one way and the other angle rapidly the other way. The sensitivity persists if the value of C_2 is varied slightly, but the estimated yield of the explosion is closely proportional to C_2.

We have now solved the equations of motion of the third ladder and fitted two observed angles with a single value of the yield of the explosion. To complete our studies of the first ladder we have to explain why three of the arms of this ladder remained straight at platform level, while the fourth had only a very slight bend.

Since the arms of the first ladder did not bend just below where the angle iron was fixed, the weakest component was the riveting of the platform to the angle iron, as was shown by the observation that the corners of the platform bent slightly. Thus, in this case, the couple C_1 was less than $2C_2$. If we write the equations of motion on the assumption that bending is occurring in the arms at the position of the top rung and also just above the platform, we should find that if $C_1 < 2C_2$, then $\ddot{\theta}_2 < \ddot{\theta}_1$, which means that bending just above the platform will not occur. Numerical substitution in the equations of motion confirms that these inequalities are satisfied. Thus, the ladder arms remain straight at platform level. We cannot therefore determine C_1, except of course that it must be less than $2C_2$, or the arms would have bent just below the angle irons. All that we can deduce from the observed angle of set (as we have already shown) is that the yield of the explosion was less than 11 kT.

There are two further mechanical points about the ladder. First, the arms of the ladders had holes through them to take the top rungs, which however were clamped to the arms by pairs of nuts on the screw threads at the ends of the rungs. To observe if there was any weakening effect, we took pieces of black mild steel strip of very nearly the cross-section of the ladder's arms and measured the yielding bending moment—when the strip was loaded at the centre and simply supported near the ends. A central hole to scale was made in other pieces, and a rod put through, with clamping nuts above and below the strip, and the yielding bending moment was measured. When the nuts were tight, the yielding bending moment was 2 % less than that of the simple specimens. When the nuts were slack, the yielding bending moment was lower by 20 %. We therefore consider that the bending moment in the ladder arms to cause yielding at the top rungs was the same as if the ladder arms had their full cross section and were clamped.

Secondly, we know from observations and from the photographs that the third ladder bent at the top rung, and that on the tall side of the wall there was a slight set at the rung next to the top. It is an interesting question why the ladder on this side of the wall did not bend at the point of fixing to the wall (the fifth rung from the top). We do not have enough information to demonstrate that the ladder would in fact behave as it did behave; but reasonable guesses about the strength of the fixings of the rungs gives the observed behaviour. The small movement of the ladder arms at the top rung on the tall side of the wall, due to elastic and plastic deformation lower down, has only an effect of adding a few tenths of 1 kT to the estimated explosive yield.

The structure of the ladders was too complicated to permit detailed calculations to be made with confidence. On the other hand, the behaviour of the ladders would have been elastic up to a certain strength of blast wave and then, with slightly more powerful waves, the deformations would have increased rapidly. In this sense, the ladders were sensitive to the strength of the blast. Our calculations indicate that the nuclear explosive yield at Hiroshima was 10 to 11 kT; and provided the basic blast and drag data are fairly accurate, this estimate should be correct within 10 to 20 %.

Prison wall at Nagasaki

The prisoner-of-war compound in Nagasaki was some 1000 ft north of ground zero. The prison was on a plateau about 60 ft above river level. A concrete wall, with very light reinforcement rods and 15 ft high, enclosed a rectangular compound. The thickness of the wall was mostly 15 in at the bottom and $7\frac{1}{2}$ in at the top, but there were variations. The wall had been cast *in situ* against a set of concrete posts 10 in × 12 in spaced at 14 ft.

The blast wave was incident at 10° off normal incidence on the north and south sides of the wall. The east and west sides were tilted; the north and south sides were in part uprooted and sheared, and in part disintegrated, by the blast; and pieces were thrown to various distances.

Figure 20, plate 6, shows a photograph of a piece of concrete from the top of the wall at the north west corner, at a distance of 1300 ft from ground zero. This piece measured 5 ft × 5 ft × $7\frac{1}{2}$ in. The horizontal distance from the original position was 80 ft and the bottom edge of the concrete was 11 ft lower than the bottom of the wall. One edge of the piece was embedded in the ground (a slight bank by the side of the road), with the edge which was the top of the wall in a nearly vertical position. The appearance of the piece embedded in the ground suggested that it had stopped where it fell, embedded like a javelin and probably rotating into the near vertical about the embedded edge.

There were many other pieces of wall scattered at various distances, but in no other case was it possible to decide where the piece had first struck the ground.

An estimate of the strength of the blast can be made on the assumption that the blast wind gave momentum to the piece of wall sufficient to carry it 80 ft horizontally while the centre of gravity was falling 21 ft. It is necessary to make certain approximations. The blast wind is assumed to have been horizontal; the blast wind while it was accelerating the piece of concrete is assumed to have been normal to the 5 ft × 5 ft face; the details of the break up of the wall are assumed to have happened quickly, and the piece then moved freely under the combined forces of the blast wind and gravity, effectively reaching the final horizontal velocity before any significant rotation had occurred.

The arithmetic works out as follows:

The duration of free flight was 1.15 s. If the explosive yield was 20 kT, the positive duration of the blast was 0.40 s, and the blast wind had done most of its work in 0.1 s. The horizontal velocity given by the blast wind to the piece of concrete was therefore about 72 ft/s. The mass of concrete per square inch exposed to the blast was 0.6 lb. The momentum given by the blast wind to the concrete was 43 pdl s in^{-2}.

If q lbf/in^2 is dynamic pressure in the blast and t_+ is the positive duration, the momentum given to the concrete by the blast in absolute units is

$$gA\int_0^{t_+} qC_\mathrm{D}\,\mathrm{d}t, \tag{29}$$

where A is the area and C_D the drag coefficient.

Using the conventional formula that the dynamic pressure varies with time as given by (2), we obtain that the horizontal momentum per square inch given to the piece of concrete by the blast is $0.22 q_0 t_+ g C_D$, where q_0 is the maximum dynamic pressure. (We could use τ instead of t_+, but the whole calculation is very rough.)

The drag coefficient is close to unity. Then

$$43 = 0.22 g t_+ q_0.$$

This equation determines the explosive yield, since q_0 and t_+ are both functions of the yield. If the yield was about 20 kT, then t_+ was 0.40 s, and q_0 was 15.3 lbf/in^2. The corresponding peak overpressure in the two shock configuration with the geometry which obtained was about 30 lbf/in^2.

A yield of 20 kT burst 1600 ft above the ground (1650 − 50 ft), would have given a peak overpressure of 32 lbf/in^2 at a distance 1300 ft from ground zero.

The numbers in the calculations can be replaced by others which seem to be equally valid. The drag coefficient was probably greater than unity, and perhaps 1.2 is a more likely value. The estimated yield is thereby decreased. The time integral of the drag is however almost certainly overestimated, since the blast pressure at high pressure levels decreases more sharply than the formula assumed. The effect of this error is to increase the estimate of the yield. Then of course there are all the other errors of unknown sizes contained in the approximations which have already been described.

To indicate the error sensitivity of the calculations on the explosive yield, suppose that our estimates of peak shock overpressure are in error by x lbf/in^2. Then the error in the yields is $0.7x$ kT, and the error in q_0 is about $7x$ %. With $x = 3$, the error in yield is 2 kT, and the error in q_0 is about 20 %.

We consider that the observation has probably been interpreted correctly and is consistent with an explosive yield of about 20 kT, although the precision is not high. If, as may have been the case, the height of burst was somewhat greater than 1650 ft, the estimate of the yield must be increased somewhat. Increasing the height of burst by 100 ft would have the effect of increasing the estimate of the explosive yield by about 3 kT.

6. Distortion or breaking of panels

A general analysis of the dynamic response of a panel which deforms plastically-elastically under an arbitrary space and time-loading function is too complicated for us to study here. The edge conditions are of course extremely important, and while a few of the examples in which we are now interested have well-defined edge conditions, others do not, and our only recourse was to make models where the edge conditions were as nearly the same as our information and our experiments would allow.

The panels on which observations were made were all found inside buildings, and although this fact introduces a complication in making allowances for the pressure–time loading curve acting on the panel, there is a substantial gain in the interpretation of the observations in that the loading built up with time, rather than being applied suddenly. In other words, the dynamic effects were less marked than they would have been with a suddenly applied load, as would have been the case if the panels had been in the open. We are therefore encouraged to make the approximation that the shape of the panel during the motion was similar to that under static

loading, and we have checked in our experiments that the shape of the panels when the loads were gradually increased remained similar. Of course, in a frictionless system where a normal mode analysis applies, the shape changes with time; and, in general, the different parts of the system are never at rest together after the time when the load is applied. In our approximation the system moves together and the shape at any time is determined by a single parameter, say the deflexion at the point of greatest deflexion. While we recognize the approximate nature of our analysis, we consider that as the dynamic effects are not going to affect out conclusions substantially, the corrections which we estimate at least give an idea of the size of the dynamic effects and give an approximate value for them.

A number of observations were made on panels which were exposed to a differential pressure on the two sides and which dished, distorted or broke. Observations were usually confined to panels where there could have been little or no leakage of air from the high-pressure side to the low-pressure side, although in at least one case a gap must have opened as the motion proceeded, and we attempt to make a correction for the leakage in this particular case. The rise of air pressure on the inside due to the motion of the panel was small, and it is easy to make a correction for this effect.

Examples which were recorded include the dishing of the top of a light steel cabinet; the dishing of the top of a heavier gauge steel tool cabinet; the dishing of two swing doors of an old, cheap type of safe, the doors meeting down the middle of the front of the safe; and the collapse of one panel of a wooden platform or stage. In every case, the damaged object was inside a building and it was necessary to relate the pressure pulse inside the building where the observation was made to the pressure pulse outside the building. The breaking of one panel of the wooden floor of a platform was different in principle from the other observations. The panel behaved elastically up to very nearly the moment when it broke, whereas the metal panels behaved elastically-plastically from nearly the start of the motion. For the time being therefore, let us disregard the case of the wooden panel and discuss the others.

Consider the example of the dishing of the top of a light metal cabinet where the door fitted well, and air leakage into and out of the cabinet during the motion, caused by the pressure inside the building, was negligible. The first part of the motion was purely elastic, but the displacements in this phase were small compared with the maximum displacements observed. The edges were not held firmly. None of the three well-defined edge conditions normally used in mathematical analyses applied (clamped edges, simply supported edges, free edges). The motion of the panel in the blast caused bending over most of the panel, and probably nowhere in the panel was tensile yielding occurring over the whole thickness of the cross-section. The integrated effects of the bending caused the shape of the panel to change. The panel certainly did not behave as a membrane or as a soap bubble. Our experiments showed that if a load was gradually applied, sufficient to cause a 'set', there was some recovery as the load was gradually removed. Thus, the motion involved both plastic and elastic deformation.

The approximate method which we use for the elastically-plastically deforming panels can at least be justified by physical arguments. Experimentally, we observe that the panels do not vary much from similarity. We only have a small purely elastic phase. The plastic forces are considerably bigger than the inertia forces. We do not have to calculate where an elastic régime passes a yield point, calculated from a curvature which is sensitive to shape.

Take coordinates (x, y, z) with z normal to the undisturbed position of the panel, and with the origin at the mid-point of the cross-section at the point where maximum displacement is to occur.

Thus, for the top of a cabinet, the origin would be at the mid-point of the undisturbed panel; for a safe door with a free edge, the origin would be at the mid-point of the free edge, before motion starts.

Then our assumption is that the displacement z of the mid-section of the bend is

$$z = f(x,y)\,Z, \quad f(0,0) = 1, \tag{30}$$

where Z is the displacement of the point of maximum displacement produced by a uniform loading $F_1(Z)$ of the panel, applied gradually.

Then at time t, the kinetic energy T_1 of the panel in its first swing is

$$\begin{aligned} T_1 &= \tfrac{1}{2}m\iint \dot{z}^2\,\mathrm{d}x\,\mathrm{d}y \\ &= \tfrac{1}{2}m\dot{Z}^2\iint f^2(x,y)\,\mathrm{d}x\,\mathrm{d}y \\ &= \tfrac{1}{2}mA\dot{Z}^2\overline{f^2}, \end{aligned}$$

where m is the mass of the panel per unit area and A is the area of the panel.

If some or all of the edges are simply supported or free, and the others (if any) are clamped and do not move, no work is done by or against edge forces. Some of the panels on which observations were made effectively come within these categories: for the others the applied load does work against edge forces, but this work is included in the total energy of distortion of the panel.

The time rate of change of kinetic energy on the initial swing is

$$\mathrm{d}T_1/\mathrm{d}t = mA\overline{f^2}\dot{Z}\ddot{Z}.$$

The rate at which work is being done against the elastic–plastic forces in the panel and against the edges is

$$\dot{Z}F_1(Z)\iint f(x,y)\,\mathrm{d}x\,\mathrm{d}y = A\bar{f}F_1(Z)\,\dot{Z}.$$

The rate at which the air pressure is doing work is $A\bar{f}P(t)\,\dot{Z}$, where $P(t)$ is the air overpressure in the building, less the air pressure built up beneath the panel.

Therefore

$$mA\ddot{Z}\dot{Z}\overline{f^2} + A\bar{f}\dot{Z}F_1(Z) = A\bar{f}\dot{Z}P(t),$$

or

$$m(\overline{f^2}/\bar{f})\,\ddot{Z} + F_1(Z) = P(t). \tag{31}$$

We can measure $F_1(Z)$ and $\overline{f^2}/\bar{f}$, and thus obtain an equation which we can integrate numerically for a given $P(t)$.

The boundary conditions are

$$Z = 0, \quad \dot{Z} = 0 \quad \text{at} \quad t = 0, \tag{32}$$

and we integrate as far as the stage where $\dot{Z}$ again becomes zero, say at time t_1 with $Z = Z_1$.

The increasing pressure inside the building pushes the central point of the panel as far as Z_1 where momentarily the panel is at rest. The elasticity of the panel then overcomes the applied pressure but as soon as the panel starts to move outwards, the elastic forces rapidly diminish. During this swing, the elastic forces in the panel may be represented by $F_2(Z)$ and the equation of motion is

$$m(\overline{f^2}/\bar{f})\,\ddot{Z} + F_2(Z) = P(t) \quad (t \geqslant t_1). \tag{33}$$

The function $F_2(Z)$ can be measured by experiment.

Depending on the parameters, the first return swing may be followed by a further inward swing, where equation (31) again applies, and by a further outward swing, and so on. We have to take the time as far as the time when the maximum pressure is reached in the building. The

rate of decrease in pressure thereafter is comparatively so slow that it may be considered to take infinitely long: and the panel recovers to some extent by its elastic component. We have to adjust the parameters so that the panel ends up at the set which we are trying to fit.

We can easily construct a simplified model which is useful in certain cases where the dynamic corrections are small.

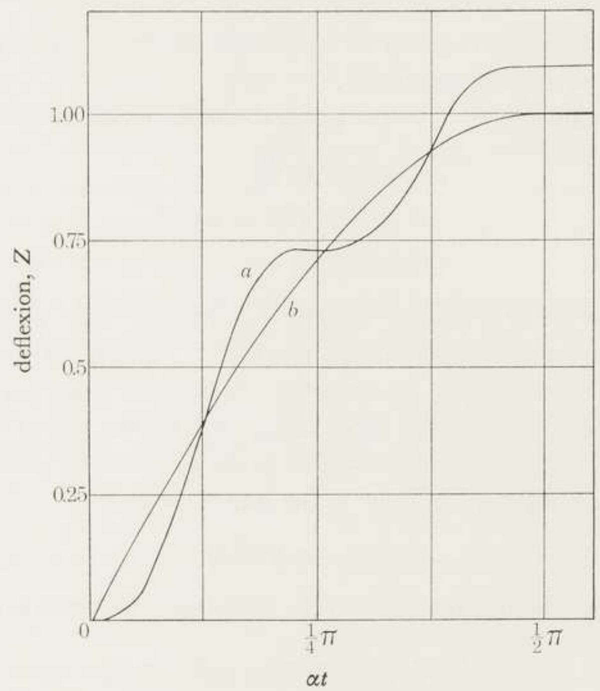


FIGURE 21. Approximate solution of the motion of a plastic panel under blast loading: (a) inertia included, (b) inertia omitted.

Let us linearize the equation of motion on the inward swings and approximate to the pressure variation in the building by a half sine wave. The equation of motion in the inward direction has the form

$$\ddot{Z}+k^2Z = A\sin\alpha t \quad (0 \leqslant \alpha t \leqslant \tfrac{1}{2}\pi), \tag{34}$$

$$Z = \{A/(k^2-\alpha^2)\}\sin\alpha t+\lambda\cos kt+\mu\sin kt. \tag{35}$$

The first swing has the boundary conditions

$$t = 0, \quad Z = 0, \quad \dot{Z} = 0. \tag{36}$$

The first swing terminates when $\dot{Z}$ falls to zero. We ignore the motion due to partial elastic recovery, and hold Z constant until a new inward swing starts when the air pressure is enough to cause further plastic deformation. Figure 21 shows the solutions, connected together for the case where

$$k = 8\alpha, \quad \alpha = 80/\pi\,\text{ms}.$$

All of the metal panels described in this section had a short wrinkle line at each of the four corners along the diagonal, closely resembling the appearance of the panels they were modelling. The wrinkle lines were caused in the models by slow steady loading: they were not manifestations of a return swing in dynamic conditions.

Tool cabinet in Nagasaki

Several tool cabinets were found in an engineering workshop. They were all similarly damaged: the tops were dished by 0.75 in relative to the edges which were not deformed, and the remaining sides were undamaged. A sample of metal was cut from one of the tops: its thickness was 0.056 in. In construction, the edges of the top panels were bent over the sides, crimped and spot-welded. We have tested statically several full-scale models of the tops using the same edge restraint and applying a uniform pressure through water into a rubber bag. From measurements of the distorted panels we found the ratio of average deflexion to central deflexion to be $\frac{2}{3}$. Each of the

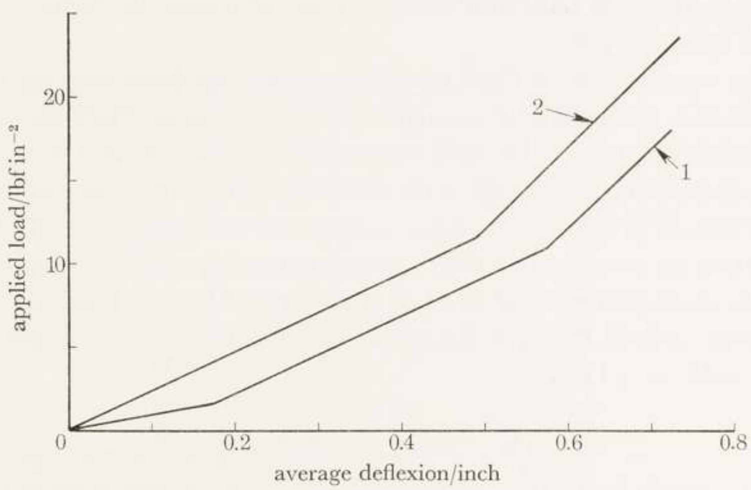


FIGURE 22. Load-deflexion curve of model of some tool cabinets in Nagasaki.

models tested showed an initial softness caused by a slight buckling of the panel. The load-deflexion curve is shown as curve no. 1 in figure 22. In quantity production, the panels would have been formed in a press and would not be buckled. The initial softness was therefore eliminated from the load-deflexion curve by measuring from an initial average deflexion of 0.15 in. The resulting curve is no. 2 in figure 22. There was some recovery when the load was removed, and it was found that a maximum load of 20.7 lbf/in^2 was required to achieve a final central set of 0.75 in. To this must be added 0.5 lbf/in^2 to allow for compression of the air in the cabinets, caused by the dishing of the top and the small elastic movement of the other faces.

The workshop was at a ground range of 1900 ft, and the building is fully documented. In all, seven photographs are available, together with plan and elevation drawings: a photograph of the building is shown in figure 23, plate 7. It was a three-storey, long rectangular building, each floor being divided into three bays by two lines of columns. The tool cabinets were on the ground floor, near the wall farthest from ground zero. The length of the building was nearly perpendicular to the radius vector to ground zero. Measurements we have made on photographs of the building show that the volume of the ground floor of the building divided by the area of the openings (V/A) was 36.5 ft. According to the method of § 3, if the hydrostatic pressure around a building with a V/A ratio of 36.5 ft is suddenly increased from atmospheric pressure to 20 lbf/in^2 above atmospheric, the ratio of the average overpressure inside the building to that outside increases to $(1-e^{-1}) = 64\,\%$ in 23 ms. If the building is immersed in the pressure pulse from a 22 kT explosion, burst at 1650 ft, then the positive duration at 1900 ft from ground zero is about

600 ms. The maximum pressure inside the building will be about 81 % of the maximum overpressure in the pulse outside the building, at time 56 ms. Model field tests showed this to be so, providing the window glass offers sufficient resistance to air flow to prevent substantial shock waves forming inside the building.

We must now calculate the magnitude of the possible error caused by the neglect of the inertia term. The 'natural' period for a swing taking the panel inwards is estimated from the loading curve to be 5.6 ms and the 'natural' period of the build up of pressure within the building is 4×56 ms or 224 ms. The maximum correction for the effect of the dynamic motion on the static area is therefore ± 2.5 % or ± 0.5 lbf/in^2. The best estimate we can make of the maximum overpressure inside the building is 21.2 lbf/in^2 and the order of magnitude of the correction due to dynamic effects is ± 0.5 lbf/ in^2.

According to the contour lines on the Nagasaki map which we have used, the building was on ground rising away from ground zero at an angle 5° to the horizontal. The effect is approximately the same as displacing ground zero by 144 ft towards the building, so that the effective distance from ground zero was 1756 ft. The change in the effective height of burst was negligible. Referring to the parametric curves of figure 2, the peak overpressure from an explosion of 22 kT burst at 1650 ft above ground at a distance of 1756 ft from ground zero is 26.2 lbf/in^2. The maximum overpressure inside the building would be 81 % of this or 21.2 lbf/in^2. Thus, our best estimate of the nuclear explosive yield is 22 kT. A change of ± 70 ft in the assumed height of burst would vary the nuclear yield by ± 1 kT.

Office cabinets at Hiroshima

More than half a dozen steel office cabinets were found in the west wing of the Army Communications Bureau at 4580 ft from ground zero. The metal thickness was 0.018 in, the cross-section was $23\frac{5}{8}$ in $\times$ 15 in and the height was between 3 and 4 ft. The method of construction and the way the top was fixed to the sides were the same as those of the tool cabinets at Nagasaki, described earlier. The tops of all the cabinets were set by 1.22 in but the other faces showed no detectable distortion against a steel ruler, except that the front and back faces, just under the top face, were pulled in $\frac{3}{16}$ in at the middle.

Two full-scale models of the top panel were tested statically and the maximum central displacement which produced a final (unloaded) set of $1\frac{1}{4}$ in was 1.5 in. The load producing this displacement, correction being made for initial softness as with the Nagasaki panel, was 4.5 lbf/in^2. To this, we must add 0.15 lbf/in^2 to allow for the rise of air pressure inside the cabinet.

The volume to opening ratio V/A for this building was 25 ft. If we take the explosion as 10 kT, then according to the methods of § 3, the blast was in the peaking region and the overpressure inside the building rose to its maximum of 92 %, of the peak shock overpressure in a time 18 ms, or say 20 ms allowing for the delaying effect of the window glass. If we assume that the tops of the cabinets responded statically rather than dynamically then we would say that the peak shock overpressure was 5.05 lbf/in^2 plus whatever correction we make for the glass in the windows. According to the approximate theory given earlier in this section, this correction adds about $4\frac{1}{2}$ %, making the peak pressure outside the building 5.3 lbf/in^2.

A nuclear explosive yield of 10 kT would have produced a peak shock overpressure of 5.1 lbf/in^2 at this site.

We must now consider what change is produced by the effects of the dynamic loading. The 'natural' period of the panel on an inward swing when plastic distortion is occurring is about

12 ms; and the 'natural' period of the build-up of pressure within the building is $4 \times 20 = 80$ ms. The ratio of the two periods is nearly 1 : 7. Therefore the panel was well into the positive swing after a complete swing. The motion of the panel is in the same direction as the load. However we make the various small corrections, we cannot escape the conclusion that the evidence of the panel is that the shock overpressure outside the building was at least 0.2 to 0.3 lbf/in^2 less than the 5.3 lbf/in^2 estimate made on the basis of no dynamic effects in the panel.

The nuclear explosive yield estimated from this observation is therefore around 9 kT; and less than 10 kT. If we had not allowed for the (small) peaking effect, the estimated yield would have been slightly smaller.

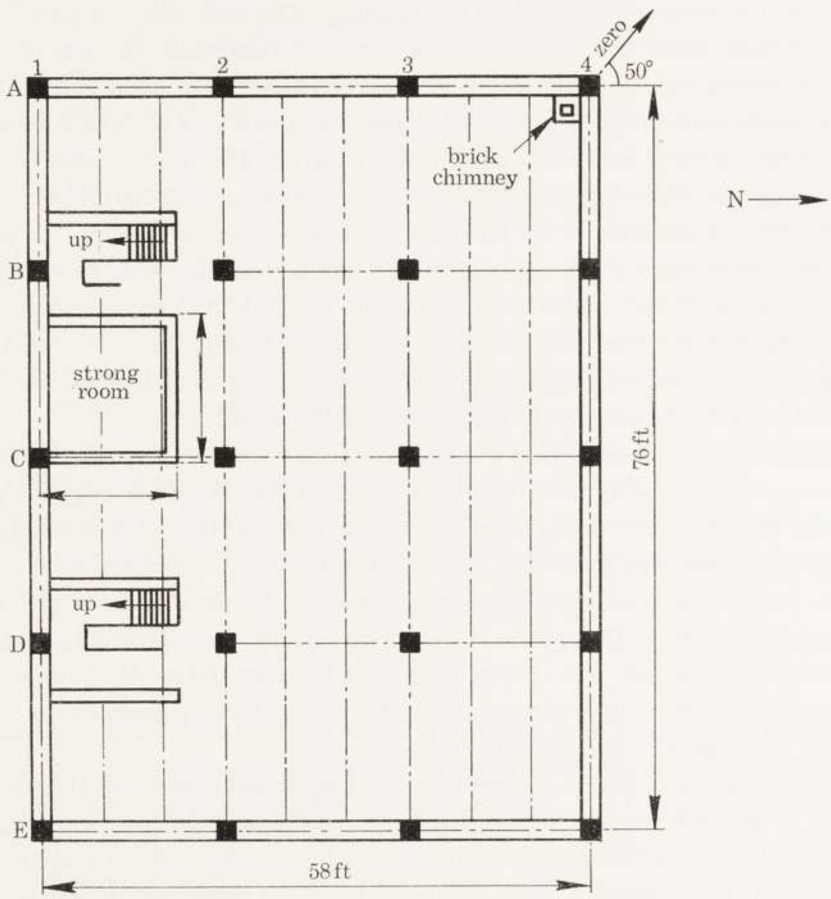


Figure 25. Ground floor plan of the Chugoku Coal Distribution Company building.

Dishing of a safe door in Hiroshima

An old mild steel safe of cheap design was found in the Chugoku Coal Distribution Company building at Hiroshima, 780 ft SSE of ground zero. A photograph of the building is shown in figure 24, plate 7, and a ground plan is shown in figure 25. The safe was standing beside the doorway in the strong-room on the ground floor, facing the window.

The two doors of the safe were dished by the air blast, the residual central deflexion being 4.0 in. There was a mortise and tenon lock and keyhole at the mid-length of the doors, and the tenon bolt of the lock was still holding the doors. The dimensions of the doors were 72.4 in $\times$ 24.7 in $\times$ 0.363 in. The doors were each attached by two hinges 10 in long, the centre of the

hinges being separated by 20 in. The abutting edges of the two doors in the centre part of the dish were separated by a little less than $\frac{1}{2}$ in, enough to permit a good measurement to be made of the metal thickness. However, the gap between the edges did not give an opening to the inside because there was a projecting steel strip attached to the inside face of the left-hand door. In the centre part of the dish, it was just possible to insert a thin 6 in steel rule into a narrow space that had opened between the edge of the strip and the edge of the other door.

Presumably there were shelves inside the safe, but these could not be located by feeling for them as far as this was possible with the 6 in steel ruler. A careful inspection of the shape of the dish, as well as running the fingers over it, revealed no ridge or change of curvature which had been caused by the resistance of shelves to buckling. The dish was completely smooth. The support to the doors given by any shelves was therefore neglected. In view of the great force exerted on the doors, any support given by shelves would have been comparatively trivial. Several structurally scaled models of the safe doors were made at a scale of 1/3 for static testing. The results of two of these tests were used in the analysis. The models were constructed from mild steel having the following tensile properties: yield stress, 37 500 lbf/in^2; ultimate stress, 47 500 lbf/in^2; total elongation, 23 %. The models incorporated the hinges, the projecting strip attached to the inside edge of the left-hand door, and the mortise lock. In one model the lock was closely fitting and in the other it was a loose fit. The only difference in behaviour was that the loosely fitting lock jammed towards the end of the unloading and prevented full recovery. It is probable that the lock on the safe doors would have been a fairly good fit, so the recovery curve for the closely fitting model was assumed to be the correct one.

The model doors were supported on a rigid angle iron frame to which the hinges were bolted, and a uniform pressure was applied by means of a water-filled rubber bag, the pressure being measured by a Bourdon tube dial gauge at the same horizontal level as the supporting frame. Eight dial gauges graduated to 10^{-3} in were used to measure the deflexion at different points on the doors. From these readings the values of the shape factors defined in (30) and (31) were found for each loading step. These were found to vary by not more than 5 % from the following values: $\bar{f} = 0.375$, $\overline{f^2} = 0.195$. The change in overlap between the model doors was measured at the quarter points, and from these measurements the area of the gap between the prototype doors was estimated as a function of deflexion.

The next problem was to obtain the pressure as a function of time inside the strong-room, and the first point to establish was whether the blast entered through the door or the window or both. It appears that there were two strong-rooms, one on the first floor directly above that in which the safe was found. Above the window of the upper room is a heavy soot deposit which is not seen at any other window. Also a metal grille covering this window can be seen to be bowed outwards. Thus it would seem that there was a strong air flow outwards through this window, and this is confirmed by the fact that the brick wall panel which contained the strong-room door failed completely. We deduce that this door was closed and the blast entering the building from the NW blew the wall in.

The open steel door of the ground floor strong-room had a heavy soot deposit on the wall above it, indicating that the main outward air flow was through the doorway. We consider it most probable that the door was open before the blast wave arrival, but it is possible that the door was closed and the window shutter was open and the door was subsequently blown open by the excess pressure inside the room.

In order to make an accurate model of the building we had to establish its dimensions. The

horizontal dimensions are marked on the ground plan shown in figure 25, and it was possible to obtain the vertical dimensions by scaling the photographs from the surveyors rod visible at the NE corner of the building in figure 24. The following dimensions were established from this and other photographs:

ground floor windows (E, N and W sides): height 9 ft 6 in; width 4 ft; intermediate columns 14 in;

first and second floor windows: height 7 ft 3 in; width and columns as for ground floor;

ground floor, floor to ceiling: height 15 ft;

strong-room door: height 6 ft 6 in; width 3 ft;

strong-room windows: 3 ft square.

Based on these dimensions, a model of the building was made from sheet brass to a linear scale of 1/92 (see figure 26, plate 8). The model was exposed at the correct orientation at a distance 8 ft 6 in from ground zero with respect to a spherical charge of RDX/TNT, 60/40, weighing 13 lb exploded at a height of 19 ft 9 in. These parameters correspond on the full scale to a nuclear explosion of 12.5 kT with a burst height of 1820 ft at a distance of 780 ft from ground zero. The pressure variations inside the model building at ground floor level were measured by flush-mounted type MQ-18 quartz piezo-electric transducers, one inside the strong-room and another in the centre of the ground floor. Similar transducers were mounted in concrete blocks which were led into the ground, one on either side of the model, to record the incident pressure–time variation. As the angle of incidence of the shock front was only 23°, these transducers were subjected to a very rapidly applied load and this excited a natural resonance at about 80 kc/s. However, this was rapidly damped and disappeared after 0.2 ms. After smoothing through the oscillation by eye the peak overpressure was estimated to be $22.0 \pm 0.3\,\mathrm{lbf/in^2}$.

The one-dimensional approximation to the equation of motion of the doors is in this case best written in terms of the average deflexion $\bar{z}$ rather than the central deflexion Z. In the nomenclature of equation (30)

$$\bar{z} = \bar{f}Z.$$

Equation (31) then becomes

$$(m/g)\,(\overline{f^2}/\bar{f}^2)\,\ddot{\bar{z}} + F(\bar{z}) = kP(t) - P_i(t), \tag{37}$$

where (m/g) has the value $2.66 \times 10^{-4}\,\mathrm{lb\,s^2\,in^{-3}}$); $(\overline{f^2}/\bar{f}^2)$ is a form factor ($= 1.4$ from the results of static tests); $F(\bar{z})$ is the load-deflexion function derived from the static tests; $P(t)$ is the time-dependent overpressure variation in the strong-room for the particular yield of 12.5 kT modelled in the field tests; $P_i(t)$ is the time-dependent pressure rise inside the safe; k is a time-independent parameter, the value of which is adjusted to give the observed permanent set at the end of the motion.

The air inside the safe will be compressed as the doors deflect, and when the average deflexion exceeds 1.5 in air will also flow in through the gap between the doors.

The front to back dimension of the safe was 24 in. Thus, when $\bar{z} < 1.5$ in

$$P_i = 14.7(1 - \tfrac{1}{24}\bar{z})^{-1.4}$$

or

$$\mathrm{d}P_i/\mathrm{d}t = 0.858\dot{\bar{z}}(1 - \tfrac{1}{24}\bar{z})^{-2.4}, \tag{38}$$

and when $\bar{z} > 1.5$ in it is necessary to add another term, $(\partial P_i/\partial t)_e$, representing the rate of rise or fall in pressure due to air moving through the gap, at constant gap dimensions.

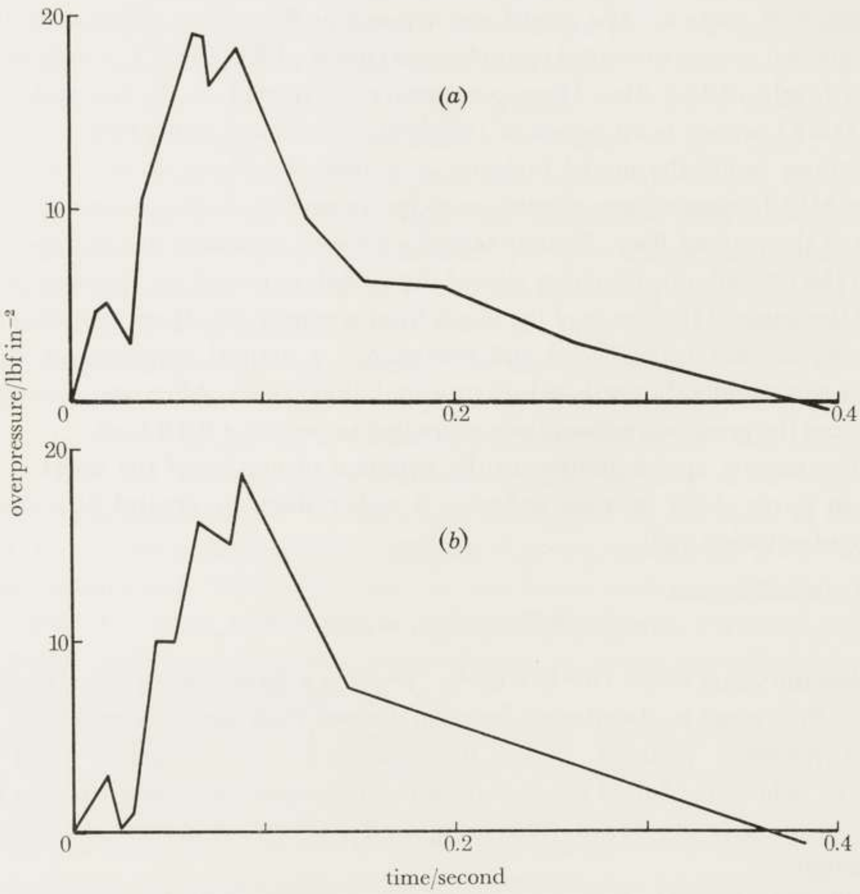
The basic equation is derived from § 3, and gives

$$\left(\frac{\partial P_i}{\partial t}\right)_e = 88\,000\frac{A}{V}(kP-P_i)\,|kP-P_i|^{-\frac{1}{2}}\,\mathrm{lbf\,in^{-2}\,s^{-1}}, \tag{39}$$

where A is the leakage area between and around the doors and V is the volume of the safe. As has been previously stated, A was determined as a function of $\bar{z}$ from the static tests. The relationship can be expressed algebraically by

$$\left.\begin{aligned} A &= 0 \quad \text{when} \quad \bar{z} < 1.5\,\text{in} \quad \textit{and} \quad t < t_1, \\ \text{and} \qquad A &= 43.6\,|\bar{z}-1.5|\,\text{in}^2 \text{ outside the above region.} \end{aligned}\right\} \tag{40}$$

This allows for the buckling at the outer corners on the return swing.



FIGURES 27 *a*, *b*. Overpressure time curves inside the model building for two cases (door open, window closed and door closed, window open).

The original volume of the safe was 79 000 in³, so V is given by

$$V = 79\,000\,(1-\tfrac{1}{24}\bar{z}). \tag{41}$$

Combining (38), (39) and (41) gives the final expression

$$\mathrm{d}P_i/\mathrm{d}t = 0.858\,(1-\tfrac{1}{24}\bar{z})^{-2.4}\,\dot{\bar{z}} + 1.13A(1-\tfrac{1}{24}\bar{z})^{-1}\,(kP-P_i)\,|kP-P_i|^{-\frac{1}{2}}. \tag{42}$$

This equation, apart from the approximations made in representing the effects of the air which has come through the gap, is not quite correct because the first term only allows for the

compression of the air initially in the safe. However, the approximation is a good one because when the $(\partial P_i/\partial t)_e$ term is present, it quickly becomes larger than the first term.

The equations were solved simultaneously using a PACE analogue computer, and adjusting the value of k to give a final value of $\bar{z} = 1.5$ in, i.e. a permanent set at the centre of the doors $Z_\infty = 4$ in.

Because of the slight uncertainty as to how the blast entered the strong-room, two curves for $P(t)$ were used. Curve no. 1, which is shown in figure 27*a*, corresponds to the more likely case of the door open and the window shutter closed, while curve no. 2, shown in figure 27*b*, corresponds to the door being closed and the window open. The values found for k were:

$$\text{curve no. 1: } k = 0.735, \quad kP_0 = 16.2\,\text{lbf/in}^2,$$
$$\text{curve no. 2: } k = 0.95, \quad kP_0 = 20.9\,\text{lbf/in}^2.$$

The computer solution for $\bar{z}$ as a function of time for loading curve no. 1 is shown in figure 28.

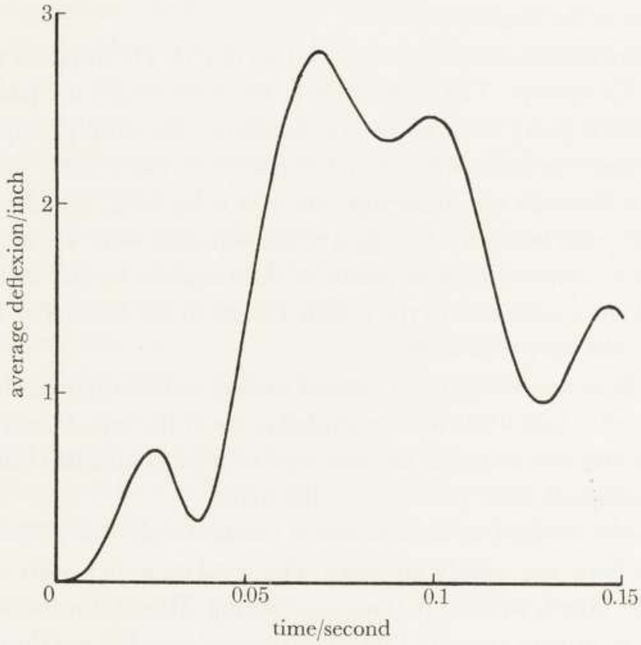


FIGURE 28. Computer solution for first case of figure 27.

Yield estimates have been made for heights of burst 1800 and 1890 ft. It is assumed that for this small range of burst heights, and small departures of the yield from the value of 12.5 kT used in scaling the model field tests, only the amplitude of the pressure loading curves changes, the time variation remaining similar in form. The estimates are:

height of burst	1800 ft	1890 ft
loading curve no. 1	8.25 kT	9.1 kT
loading curve no. 2	11.7 kT	13.1 kT

As explained above, we think that the height of burst was 1890 ft and that loading curve no. 1 is the correct one (door open). The observation therefore gives the nuclear explosive yield as 9.1 kT. On the other hand, it is possible that loading curve no. 2 is the correct one (door closed), in which case the best estimate of the nuclear explosive yield is 13.1 kT.

Wooden platform floor in Hiroshima Telephone Exchange

The Telephone Exchange 3450 ft WNW of ground zero in Hiroshima had a wooden platform or stage 166 in × 192 in erected as a superstructure on part of the main ground floor, which was of concrete. The floor boards were tongued and grooved. Everything was of high quality, with excellent workmanship. There were no openings and there could have been very little ingress of air in the transient blast conditions.

The purpose of the platform was not discovered but the wooden floors were covered by mats at the time of the explosion.

The joist underneath one of the panels of the floor had broken, and even in the configuration in which they were found, had not made any openings for air to pass. The floor was partly taken up in order to discover the details of the construction. Figure 29, plate 8, is a photograph taken when the floor was being dismantled. The damage at the left-hand end of the panel which failed was made in taking up the floor. The blast damage can be seen in the dish under the long thin piece of wood, put across the floor.

There were in all four panels, only one of which had failed. The support was provided by nine equidistant joists at 17 in centres. The outer ends of the joists under the panel which failed were resting freely on the brick party wall, and were therefore only simply supported. The joists ran from this wall over a concrete beam 4 in × 6 in and then over two similar beams and finally were inset with cement into holes on the inner top side of a brick wall, based on the concrete floor. The free lengths of the joists between the edges of the supports were 41, 39, 39 and 41 in respectively, and the distance between the mid-points of the concrete beams was 43 in. The 6 in sides of the concrete beams were vertical and the beams rested on the floor. The fit of the floor boards everywhere was snug and very well done.

The covering boards at one end were attached to and were supported only by joist 1, while the other ends were fixed to joist 9 but were extended to cover the top of the wall built on the floor. Thus the floor boards did not transmit the full load to joist 9, and this effect spread to joist 8; but all of the other joists took their full share of the load.

The observations were thought to be significant because only one panel had failed, and the space underneath the floor was nearly air tight. The weakest panels were of course the two end ones, and only one had failed, without making an opening. Hence it was considered that the load on the platform was very little in excess of that which caused one, but not the other, panel to break.

The dimensions of the joists were taken, and the distance of the break taken from the simply supported end. The exact point of break could not be exactly defined; measurements were made to a point which appeared to be central on the break.

Beginning with the joist which was close to the main wall of the building and which was simply supported at each end, and ending with joist 9, which was close to the front wall, the following measurements were made:

joist number	depth/in	width/in	point of break/in
1	$2\frac{1}{16}$	2	22
2	2	2	14
3	$2\frac{1}{16}$	$2\frac{1}{8}$	14
4	$2\frac{1}{16}$	$2\frac{1}{16}$	12
5	$2\frac{1}{16}$	2	14
6	$2\frac{1}{16}$	$2\frac{1}{16}$	15
7	$2\frac{1}{16}$	2	14
8	$2\frac{1}{16}$	2	not broken
9	$2\frac{1}{8}$	$2\frac{1}{16}$	not broken

Hence the average dimensions were $2\frac{1}{16}$ in $\times$ $2\frac{1}{16}$ in, and the point of break was 14 in from the simply supported end, except for the end joist (joist 1) which was simply supported at both ends and broke in the middle. The two joists which did not fail were those nearest the front wall. The load on the joist against the wall was obviously only about half of that on the other, and this one took some of the load from the next one.

Inspection of the panel suggested that part of the floor shown at the left-hand side in the photograph had been taken up at some time (notice that the floor boards are cut) and part of the joist taken out and replaced by another piece. This joist was only simply supported at both ends. All of the joists except the one along the front wall had also snapped over the nearest concrete beam.

A piece of one of the broken joists, and parts of the floor boards, were taken back to England for tests which were made by the Forest Products Research Laboratory at Princes Risborough.

The Forest Products Laboratory stated that the wood of the joist was slightly cross grained but they were able to obtain two similar straight grain samples for test. Young's modulus was 1.8×10^6 lbf/in^2 and the bending moment to break a straight grained 2 in $\times$ 2 in cross-section was the average of 14 050 and 17 280 lbf in. Allowing 10 % for the fact that joist 1 was slightly cross grained, the bending moment to snap a $2\frac{1}{16}$ in $\times$ $2\frac{1}{16}$ in cross-section was 15 400 lbf in.

Let us first calculate what static load would just break a panel. We need the solution of the elementary problem of a uniformly loaded beam of length $2L$, simply supported at the left-hand end, freely supported at the middle, and clamped at the right-hand end. (The actual panel reflects this pattern about the clamped end.) The maximum bending moment occurs at $11L/28$ from the left-hand end and has the value $0.0774WL^2$. The dynamic loading problem is not quite the same but the breaking point would be expected to be at about the same place. The observed break was at 14 in from the simply supported end, compared with 17 in in the theoretical static case.

The observation that the panel (just) broke can be interpreted as meaning that the joists (just) broke under the load applied by the blast over a space between two joists. Statically, therefore, if a uniform loading pressure P lbf/in^2 just broke the panel, the load was $17P$ lbf per inch run. Therefore

$$15\,400 = 0.0774 \times 17P \times 43^2,$$

$$P = 6.35\ \mathrm{lbf/in^2}.$$

We must now consider how the dynamics of the motion may be taken into account. The most important consideration in this case is that the air pressure below the platform varied considerably as the joists and boards were pressed downwards. The 'natural' period of the platform was increased by the effect of the air in the space below the platform, and the 'air cushion' below the platform took up a substantial part of the load being applied on the top surface.

When the joists were on the point of breaking, the average depression was 1.2 in and the greatest depression was 3.2 in. The rise in pressure in the air below the platform, due to adiabatic compression, was 3.2 lbf/in^2. With this rise in pressure taken into account the natural period of the platform was 21 ms.

The air pressure in the building would have risen to its greatest value in 40 ms, and the greatest value would have been 85 % of the peak shock overpressure in the air outside the building. The 'natural' period of the pressure rise in the building was therefore 160 ms, and the ratio of the natural period of the platform to this period was 21 : 160, or approximately 1 : 8. The greatest pressure that the platform could stand was therefore 8/9 of the maximum possible static load, it

being understood that the pressure of an air cushion underneath must be included. The pressure that could have just broken the panel is therefore 8.5 lbf/in^2, and the peak shock overpressure was 10.0 lbf/in^2.

A nuclear explosive yield of 10 kT would have given a peak shock overpressure of 8.0 lbf/in^2; and 16 kT would have given 10.0 lbf/in^2.

The observation on the platform apparently suggests that the nuclear explosive yield was 16 kT, but we believe that we have not interpreted the observation correctly.

Joist number 1 was not continuous. As already mentioned, the part that straddled the panel section which broke was a separate piece. Joist 1 was more flexible than the others and more load was thrown on joist 2, which failed. Joist 1 would have taken about 0.3 of the load on the space between joists 1 and 2; joist 2 would have taken the other 0.7 plus 0.5 of the load between joists 2 and 3. So a load of barely 8.3 lbf/in^2 would have broken joist 2. The failure would spread progressively across the platform, and in due course the joists broke again over the concrete beam.

We calculate that a nuclear explosive yield of the order 10 kT seems to be capable of explaining the observations on the platform, but the precise way in which the damage occurred is debatable.

There were two other wooden platforms on the Telephone Exchange, but these were obviously nowhere near critical. They had failed badly, and no estimates could be made from them.

7. Crushed cans and drums

Many empty and partially squashed drums were found in the two cities, and a few of those found appeared to offer prospects of permitting an estimate of the blast to be made. Sometimes the opening of the can was large, and any such can found in a building could not be used to make estimates. Sometimes the can or drum was full of liquid, and had therefore not collapsed even though the overpressure was amply sufficient to collapse the can or drum if it had been empty. Often the can or drum had been in a severe fire and the damage may have been caused as much by the fire as by the blast. Nevertheless, among the hundreds of cans and drums which were examined, a few appeared to have been crushed in a significant way by the blast. In this Section we discuss the interpretation of these observations.

The basic idea of the three observers in the initial stages was that it should be possible to obtain upper and lower limits to the peak pressure on the go–no go principle. If an empty can with no opening or only a small opening had collapsed, the peak pressure could be given a lower limit by measuring (at a later date) the pressure that this type of can could withstand. Similarly, if an empty can with little or no opening had not collapsed, the peak pressure in the blast could be given an upper limit. This idea required a search for cans or drums which were fairly common, and the obvious type were 46-gal (imperial) petrol or oil drums and 4-gal (imp) petrol cans. While it was not known at that time whether the observed loss in volume of a can due to compression would prove to be interpretable, estimates were made on the spot of the amount of compression of cans or drums which were squashed and which had an opening of sufficient size to relieve the internal air pressure as the blast wave diminished.

Four-gallon cans

There were many hundreds of new empty rectangular 4-gal (imp) cans with their filler caps in place 6400 ft from ground zero at Nagasaki, in the depot of the Standard Oil Company on the water front, south of ground zero.

There were a large number of empty new and used 4-gal cans with their filler caps in place, 5700 ft north of ground zero at Hiroshima, at the Standard Oil depot.

There was a dump of empty used 4-gal cans in the Nagasaki railway yard, 7600 ft south of ground zero.

Most of the cans at the Standard Oil depot in Nagasaki were in a slightly squashed condition, caused by the blast. Some (perhaps one in five or one in ten) were in perfect condition. There was no obvious explanation, such as shielding, to account for the different behaviour of the cans. The maximum average dishing over the four faces of the most distorted cans was 0.4 ± 0.1 in (a maximum of 10 % loss of volume). Four cans were cut up, and the thickness measured. The thickness was uniformly 0.013 in. The can dimensions were $9\frac{1}{4}$ in $\times$ $9\frac{1}{4}$ in $\times$ $13\frac{3}{4}$ in.

The cans at the Standard Oil depot at Hiroshima had behaved in exactly the same way. Possibly a very detailed statistical set of measurements would have revealed small differences from the cans at the Standard Oil depot at Nagasaki, but such an examination was not made. All of the cans at the Nagasaki railway yard appeared to have survived the blast without being dented, or else they re-inflated perfectly (which seems unlikely).

Experiments which subjected similar cans to a static external pressure, achieved by using a suction pump to remove air from inside, showed that the Japanese cans could withstand a static pressure of 1.2 to 1.5 lbf/in^2. The fact that some cans at the Standard Oil depot in Nagasaki were collapsed by 10 %, giving an internal pressure of 2.1 lbf/in^2 above atmospheric, suggests that the external pressure might have been about 3.3 to 3.5 lbf/in^2. However, if the external blast pressure had been greater, the can would have collapsed and then partially re-inflated. Similar considerations apply to the cans at the Standard Oil depot in Hiroshima.

The survival of all the cans at the Nagasaki railway yard suggests that at 7600 ft from ground zero the peak pressure was little more than 1.2 lbf/in^2. This is less than half of the value that would be predicted from the basic data. A low value would seem to be well established. The probable explanation for the low value is a combination of reasons—scattering of the blast by buildings, reduction in the blast by the damage being caused, and an upward refraction of the blast near the ground in a warm layer near the ground due to heating of the ground by the strong sunshine.

Forty-six-gallon drums

The three Manhattan District observers inspected a large number of empty 46-gal (imp) drums in the two cities. In some cases the openings into the drums were closed by stoppers or caps, and in other cases one or two openings were open.

Only two types of drum were considered to offer any hope of providing a significant reading. These were the types of 16 B.S.W. gauge (0.064 in thick) and 18 B.S.W. gauge (0.048 in thick). There were some drums of heavier gauge but none of these showed any signs of failure.

Our description of the observations and their interpretation will be clearer if we first describe experiments we have made on 46-gal drums to measure some of their mechanical properties and see how reproducible they were. We used only drums of British or American manufacture, both recent and made in 1945/46.

Drums with the openings completely closed were subjected to a steadily increasing hydrostatic pressure in a compression chamber, the compressing fluid being sometimes air and sometimes water. Some of the lighter gauge drums were also subjected to a collapsing pressure by partial evacuation.

The hydrostatic pressure which caused failure varied appreciably from drum to drum. The results are too lengthy to give in full and an important conclusion is the variability. We may summarize as follows.

Drums of 18 B.S.W. when slowly evacuated collapsed at 7.6, 8.0, 8.4, 8.8 and 9.3 lbf/in^2; but two drums withstood 13.6 lbf/in^2 without collapse. Two more drums, each loaded with 500 lb of sandbags over the curved side, collapsed at 8.0 to 8.5 lbf/in^2. Drums of 16 B.S.W. failed in a three-node collapse at 20, 22, 25 and 30 lbf/in^2, while another modern set with a very good end rolled joint all failed in a two-node collapse at 25 to 35 lbf/in.^{2} (These modern drums were definitely stronger than those of the 1945 period.)

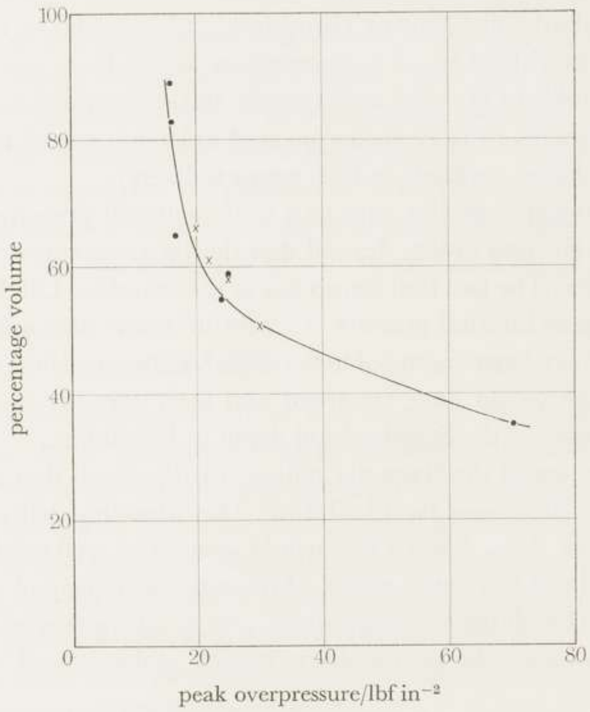


FIGURE 30. The percentage volume of empty 46-gal 16 B.S.W. drums collapsed by a known applied external pressure difference.

Drums which were slightly dented before test appeared to lose strength by 5 to 10 %.

The conclusions are that 18 B.S.W. gauge drums will withstand a hydrostatic pressure of up to 8 lbf/in^2, but that at this pressure some drums will fail. The 16 B.S.W. gauge drums, especially used drums of Japanese war-time manufacture, will withstand up to about 20 lbf/in^2, but some will fail at this pressure.

We have also subjected 16 B.S.W. gauge drums to the blast wave from a nuclear explosion. Ingress of air was prevented by fixing a flap over the main opening by means of Cellotape. As soon as the external pressure fell below the inside pressure, the flap opened and air escaped. Thus there was no re-inflation.

Figure 30 shows the measured volume per cent of the 16 B.S.W gauge drums squashed by a nuclear blast wave plotted against the peak hydrostatic pressure. Also shown plotted against the collapsing pressure in the same figure are the final volumes after collapse of some 16 B.S.W. gauge drums, put in an air-compression chamber. (The drums did not re-inflate when the

pressure was slowly released because these sudden large collapses caused the rolled joints at the ends to leak.)

A curve has been drawn by eye to represent the experimental points. It will be observed that, although the pressure to induce collapse in a drum has been shown to vary considerably from drum to drum, the stronger the drum is against collapse the more it collapses when it does fail. Indeed, the experimental results seem to show that the pressure which caused collapse can be calculated, within a few parts per cent, as the sum of the pressure increase inside the drum caused by adiabatic collapse and a constant (8 lbf/in^2) representing the strength of the drum in the collapsed state. It is as if most of the work done by the collapsing pressure goes into plastic deformation of the drum rather than into kinetic energy of the drum walls.

We now give the observations in the two Japanese cities on 46-gal drums, and seek to interpret them.

A reconditioned partially collapsed empty 46-gal 16 B.S.W. drum was found by the three Manhattan District observers just inside the north-west wall surrounding the Commercial Display Hall in Hiroshima. The drum was 650 ft from ground zero. The bleed hole plug at one end was in place, but the filler cap in the middle cylindrical panel was missing. A photograph of the drum is shown in figure 31, plate 9.

Measurements on the drum were made, and the cross-sections at several heights were sketched. The collapsed volume of the drum was estimated on the spot to be 65 % of the original volume. There may be an error of a few parts per cent in this estimate, but the error if any is such as to make the true volume greater than 65 % of the initial volume. This opinion is based on comparisons between the photographs of the drum and actual drums collapsed to a known percentage on a three-node failure configuration.

The peak hydrostatic pressure in the blast, according to figure 30, was 20 lbf/in^2. We have made a calculation (not given here) to demonstrate that the air pressure inside the drum would be released sufficiently quickly through the filler opening, as the blast pressure decreased, to prevent the drum from re-inflating. With a height of burst of 1890 ft the corresponding yield of the explosion was 12.0 kT.

We can get some idea of the accuracy of this estimate from two lines of argument. First, if we took the estimate 20 lbf/in^2 for the peak overpressure and used the basic blast data in the U.S.A.E.C. book, we get the nuclear explosive yield as $10\frac{1}{2}$ kT instead of the 12 kT obtained from our data. Secondly, we note that the drum was reconditioned and must have had some dents from usage (thus slightly weakening the strength against collapse). The collapsed volume was probably a little greater than 65 %. An inspection of figure 30 suggests that the peak overpressure might have been as low as 17 lbf/in^2 and that any value greater than 22 lbf/in^2 is very improbable. Changing the peak overpressure by 2 lbf/in^2 is equivalent to changing the nuclear explosive yield by 1 kT.

Thus, the observation suggests a nuclear explosive yield at Hiroshima of 12 kT. The error should be small, and 12 kT is more likely to be on the high side than on the low side.

In Nagasaki, the three Manhattan District observers did not find any collapsed 16 B.S.W. gauge 46-gal drums, but they made several observations on 18 B.S.W. gauge drums. The search started at the Standard Oil depot 6600 ft from ground zero where there were a large number of drums, some empty and some full. None showed any signs of failure. The observers then proceeded northwards, looking systematically for drums in the Mitsubishi Arms Factory. Half a dozen empty 18 B.S.W. gauge drums standing in the open at 5000 ft from ground zero were not

collapsed. The first collapsed drums were found in the more northerly building of the Ordnance Plant, 4270 ft from ground zero. The observers recorded that some of these drums had collapsed but that a substantial proportion of empty drums had not collapsed.

A mixed dump of 46-gal drums was also found at the Torpedo Factory at 4500 ft north of ground zero. None of these drums showed any signs of collapse.

The best way of interpreting the above observations is not obvious, but one deduction seems secure. The least external static pressure observed to cause failure of a 18 B.S.W. gauge 46-gal drum was 7.8 lbf/in^2. The least peak hydrostatic pressure in a long duration blast wave, such as produced by a nuclear explosion, which would cause collapse of the weakest 46-gal 18 B.S.W. gauge drum must be slightly less than 7.8 lbf/in^2, because of the additional effect of wind pressure. The least distance from ground zero at which drums were observed not to have failed was 4500 ft. Hence, the peak blast pressure at 4500 ft was less than 7.8 lbf/in^2. The yield was therefore less than 29 kT.

The fact that at 4270 ft some drums had collapsed and some had not, taken in conjunction with the results of the static compression tests, suggests that the collapsing effect of the blast wave was the same as that of a static pressure of about 8.5 lbf/in^2. The scanty evidence which we have about the effects of blast waves on drums is consistent with the view that the equivalence between static pressure and peak blast pressure is obtained by equating the static pressure to the peak blast pressure plus half of the dynamic pressure. With this assumption, the peak blast pressure at 4270 ft was 7.9 lbf/in^2. The error might be 0.5 lbf/in^2. The explosive yield would therefore fall between the limits of 20 and 28 kT.

The evidence of the 18 B.S.W. gauge drums at Nagasaki is that the yield of the explosion was about 24 kT, with a possible error of a few kilotons.

Can at 3100 ft in Nagasaki

A nearly cubical can of a gauge similar to that of a 4-gal petrol can, and having about the same volume, was found with the filler cap missing in the large building immediately north of the Takenobukomachi Bridge. The distance was 3100 ft from ground zero. A photograph of the can is shown in figure 32, plate 9 (the 46-gal drum in the background was full of water).

Measurements of the can were made on the spot, and the compressed volume was estimated at 65 % of the original volume. This estimate was probably a good one because the simple initial shape of the can enabled the dimensions to be measured with good accuracy.

At a test of a nuclear device of about the same yield as the Nagasaki explosion, nine empty 4-gal cans were exposed at the 12 to 15 lbf/in^2 level. If the can found at Nagasaki had behaved in the same way as the 4-gal cans, a final volume of 65 % would correspond with a peak pressure in the blast of 12.2 lbf/in^2. An explosion of 20 kT burst at 1650 ft would give 11.9 lbf/in^2 at 3100 ft from ground zero. Thus, the can found at Nagasaki suggests a yield of about 20 kT or a little more. However, the cans tested at the field trial of a nuclear device varied in compression by ± 6 % of the initial volume from the mean compressed volume. The value of the peak pressure deduced from the can found at Nagasaki could therefore have an error of about ± 1.5 lbf/in^2. The limits on explosive yield would then become 16.5 kT (10.7 lbf/in^2) and 32 kT (13.7 lbf/in^2). The yield varies considerably because of the knee-shape of the parametric peak pressure curves. The observation is therefore not of much value, but it does fix the yield as not less than 16.5 kT, and indicates a best value of the yield as 20 to 25 kT.

Blue-print container: Hiroshima

Figure 33, plate 9, shows a photograph of a blue-print container found in the Communications Bureau, 4580 ft NE of ground zero. The container was of light construction but it was well made. The lid swung open or shut on a hinge, and the support for the other arm of the hinge was a light piece of metal fixed to the top of the body of the container. The lid was cramped into position by means of a handle, held by a rivet to the lid. The handle swung to and fro, thus moving the lever arm in or out of position under a flimsy catch, fixed to the top of the container. The lid had a good quality rubber washer round the conical shaped inset wall, built into the top of the container. The end of the lever arm had been forced upwards and the catch was badly bent. This damage was presumably caused by the air pressure inside the can being greater than the outside air pressure when the pressure outside was falling back to atmospheric or below. The lid must have come out with some violence, because the top of the main body of the container was bent where the hinge support was attached.

Four of these containers were found in the Communications Bureau. Three had been in a fire, and had burnt paper ash inside, but all four appeared to be equally squashed. The one that appears in the photograph was clean and empty and had not been in a fire. This particular one was taken to England for test.

The results were as follows: original volume, 57 130 cm^3; volume in collapsed state, 51 530 cm^3; adiabatic pressure above atmosphere, 2.28 lbf/in^2; further collapse in collapsed state at 0.81 lbf/in^2.

The wall material was so thin that dynamic corrections are negligible. If the motion of collapse was 'dead beat', then the maximum air pressure acting on the container was 3.1 lbf/in^2 above atmospheric. This value may be misleading for the following reasons. The container, in its collapsed state was still a 'springy' structure, i.e. an external pressure applied to the container in its collapsed state led to an elastic reduction of volume before further plastic deformation occurred. Experiments to determine the elastic régime were not made.) Secondly, there may have been some plastic reflation of the container when the air pressure inside became greater than the pressure outside and while the lid was being blown out.

The blue-print container was in the building (Communications Bureau) where observations were also made on the dished tops of some office cabinets (see § 6). The maximum pressure in the building, according to the evidence of the office cabinets, could barely have reached 4.5 lbf/in^2, which equals the pressure expected within the building from a nuclear explosion of 10 kT, provided the blast is not diminished by scattering and doing damage. The evidence of the blue-print container, taken at its face value, indicates a lower peak pressure level and nuclear explosive yield, but a partial explanation is that we do not have enough information to estimate exactly how the blue print container behaved as the blast pressure diminished.

8. Recapitulation of yield estimates and best values

We recapitulate our estimates of the nuclear explosive yields and present the values in tables 8 and 9. The order in which the observations are given does not follow the section number, but has been chosen according to the distance from ground zero. The yield estimates have all been made in terms of an explosion over bare ground, whereas the mechanical damage done by the blast and the scattering of the blast by buildings in the two cities must to some extent have reduced the blast waves as the waves spread.

We have no statistical method of assessing the weight to be attached to the interpretation of the various observations, but we have divided them into four categories according to our opinion. The categories have been marked as good (g), fairly good (f.g.), fair (f), and poor (p); and the comments column calls attention to any special point.

TABLE 8. HIROSHIMA

observation	distance GZ/ft	yield/kT	reliability	comments
collapsed oil drum	650	12	g	should be a good estimate; more likely to be high than low
dished safe doors	780	9.1 13.1	f.g. f.g.	which is the fairly good estimate depends on whether the strong room door was open (9.1, more probable) or closed (13.1, less probable)
no wooden power line snapped	1000 to 2500	< 14 if Nagasaki was not more than 20	g	mechanical properties of wood not known accurately. Inequality reliable but no evidence that it is a close limit
bent I beam poles	1300	12	g	should be a close estimate
overturned memorial stone	1650	$> 8\frac{1}{2}$	g	the most impressive of the stones overturned but no evidence that the stone only just overturned
bent lightning conductor	2200	14 ± 4	p	discrepant? Observation good but blast and drag data in difficult regions and are not reliably known
bent lightning conductor	3080	10	g	should be a close estimate
bent steel ladders	3200	10 to 11	f	structure too complicated to allow careful analysis
broken panel in stage platform	3400	~ 10	p	platform almost certainly failed progressively
overturned memorial stones	4280	8	g	a well-defined observation, shielding effect of surrounding houses taken into account, yield apparently falling
collapsed blue print container	4580	peak overpressure down 30%	?	may have been some elastic recovery and/or some reflation; yield falling?
dishing of tops of office cabinets	4580	9	f	yield falling?
10 to 20% of empty 4-gal petrol cans undamaged	5700	peak over-pressure down by about half	g	clear evidence that the blast was less than it would have been from an explosion over an open site

Among the observations in Hiroshima, we think that the collapsed oil drum at 650 ft from ground zero (indicating 12 kT), the bent I beam poles at 1300 ft from ground zero (indicating 12 kT), the bent lightning conductor pole at 3080 ft from ground zero (indicating 10 kT) give the most reliable estimates of the nuclear explosive yield. The basic data were known or were obtained with good accuracy and the observations all lead substantially to the same value. The nil result on the non-snapping of power line poles is general supporting evidence, and so is the evidence of the safe doors. Most of the other observations add little except that there is a clear indication that at ground level the blast wave at distances exceeding 3000 to 4000 ft was progressively less powerful than it would have been over bare ground, due to the scattering of the blast by buildings, by shielding and by the damage being caused.

There is one indecisive result. The lightning conductor pole on the main roof of the Chugoku Electric Company Building at 2200 ft from ground zero, and the other pole near the cupola on

this building, shown in figure 8, indicate the nuclear explosive yield as 14 ± 4 kT. The basic blast data, the drag coefficient and the enhancement of the blast wind by the building are all involved. It seems clear that the Mach number for the pole near the cupola began at a value where the drag was rising very fast with Mach number: there is no other variable which would explain why the pole near the cupola was bent so much more than the other pole. We do not know exactly how to explain why the nuclear explosive yield is 20% higher than that obtained from the other observations. We think that we have used some incorrect basic data and that the nuclear explosive yield was not as high as 14 kT.

Table 9. Nagasaki

observation	distance GZ/ft	yield/kT	reliability	comments
throw of piece of prison wall	1300	~ 20	p	dynamics probably complicated
dishing of tops of tool cabinets	1900	22	f.g.	many cabinets behaved in the same way; edge support conditions copied but may not have been exact under load
collapse of 4-gal can	3100	$>16\frac{1}{2}$ <32	g	reliable but widely separated limits
snapped power line poles	3300	$\geqslant 20$	g	yield of wood not known accurately but checks with observations in Hiroshima
collapse distance of 18 B.S.W. oil drums	4270 to 4500	>20 <29	g	reliable but limits too wide to be of much value
overturning of memorial stones	4610	$\geqslant 19\frac{1}{2}$	g	should be a close under-estimate;
	5430	>19 to 21	g	density of stone not known accurately
10 to 20% of empty 4-gal petrol cans undamaged	6400	peak over-pressure down by about half	g	clear evidence of reduction of blast by the damage caused and by scattering
no damage to empty 4-gal petrol cans	7600	over-pressure not much over 1 lbf/in^2	g	clear evidence of reduction of blast

The value of 12 kT for the Hiroshima explosion, obtained from the bent I beam pole at 1300 ft is a very important observation because it gives clear evidence that the yield was about 12 kT. The evidence of this pole was supported by the evidence of four similar nearby poles; the three nearer were bent slightly less and the one farther away slightly more. The particular pole considered was in an open site and the effect of the nearest substantial bodies (the trees) would have been slightly to deflect the blast wind to make the load on the I beam a little greater. However, the effect would have been delayed by 10 to 20 ms and is probably insignificant. If the value we have used for the drag force resolved normally to the plane of the web ($1.96qh$) is in error by more than 10% or so, we would probably make a change in our best value of the nuclear explosive yield at Hiroshima but a large change seems unlikely.

Turning now to the results from Nagasaki, the interpretation of the observations clearly points to a nuclear explosive yield of a little more than 20 kT. The estimates agree closely but this is partly a coincidence because the precision which could be obtained reliably was not high. The best value appears to be 22 kT.

We cannot determine a statistical mean and a probable error for the explosive yields at Hiroshima and Nagasaki by normal techniques. However, we can say that the results of our work show that the Hiroshima explosion had a fairly high probability of being between 11 and

13 kT (we would say not less than a 50 % chance). The explosion at Nagasaki had a similar probability of being between 20 and 24 kT. Recalling that in conventional statistical terms, there is a 50 % chance that the error of the mean is not more than the probable error, we may summarize our conclusions by stating that the nuclear explosion at Hiroshima was 12 ± 1 kT; and the nuclear explosion at Nagasaki was 22 ± 2 kT.

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9. Appendix

The dynamic pressure

Obviously, the reliability of our work depends on the reliability of the basic blast data. In § 3 the basic data have been summarized and discussed. There is a further point to be considered. The experimental records of the blast waves in several tests of nuclear explosives, over the range of peak overpressures 5 to 10 lbf/in², conform closely to equation (1). However, we have no experimental records of the dynamic pressure q. We have assumed that the dynamic pressure, being nearly quadratic in the overpressure, therefore shows a time variation which is quadratic in the time variation of the overpressure, i.e. equation (2). The question should be asked: accepting the experimental evidence that (1) is accurate, how accurate is (2)?

In a one-dimensional travelling sound wave, the overpressure $p(t)$ is directly proportional to the mass velocity $u(t)$; but in an expanding spherical wave, such proportionality only applies at large distances from the origin. Moreover, in our case, the waves are finite and the peak overpressure p is usually of the order one-half or two-thirds of the atmospheric pressure p_0.

We can get a power series solution in $p(t)/p_0$ by using Riemann's theory of travelling one-dimensional finite waves, described for example in chap. x of Lamb's *Hydrodynamics*. The theory needs extension for our application, and the most convenient formulation for our purpose will be found in a review article by Penney & Pike (1950).

The motion is assumed to be plane, cylindrical or spherical and is defined by two functions R and S. Neglecting the change of entropy across the shock front, as is permissible in the situation of present interest, the function S is streaming inwards and away from the shock front, and its value as it leaves the shock front is zero. Hence at distance x from the centre of the wave system, at a point within the blast wave, the mass velocity u is very nearly equal to the function σ, which is $5(c-c_0)$, c being the local velocity of sound at x determined by the ambient pressure and unchanged entropy. We can thus relate $\frac{1}{2}\rho u^2$, the dynamic pressure $q(t)$, to the overpressure $p(t)$ and obtain a power series expression for this in terms of $p(t)/p_0$. We find that $q(t)$ has exactly the form given in (2), except that there is a multiplier.

$$[1+\{p-p(t)\}/7p_0]. \tag{43}$$

The effect of the pulse being finite, rather than infinitesimal as in sound, is that the formula (2) for $q(t)$ gives the correct starting value, has its zero at the right time, has a starting slope 2 to 5 %

too large, and values near time t_+ up to 10 % too small. The time integral of the dynamic pressure over the positive phase of the pulse in our circumstances is 3 % too large for $p = 10\,\mathrm{lbf/in^2}$.

The correction for radial expansion can also be estimated. In our applications, the pulse is effectively expanding spherically about the image reflexion point in the ground of the explosion point, and $\kappa = 2$. At time t after the shock wave has passed a point distance r from ground zero, the S point at r came from the shock front at time $\frac{1}{2}t$, when the shock front was half as far away from r as it was at time t. Hence the value of S is approximately

$$-(c_0 u_m t_+/r)\,(1-\mathrm{e}^{-2t/t_+}), \tag{44}$$

where u_m is the mass velocity at the shock front. The approximate value of u given by $5(c-c_0)$ must therefore be increased by the negative of the term (44) to allow for the effect of the 'spherical' expansion.

To take an example (and neglecting the peaking in the blast), the shock overpressure in Hiroshima at distance 3550 ft from ground zero was about $8\,\mathrm{lbf/in^2}$ and the mass velocity was 375 ft/s. The positive duration was 0.59 s. When the overpressure fell to zero, the mass velocity was 54 ft/s. If a simple Friedlander function were fitted to the mass velocity or equation (2) to the dynamic pressure, the positive duration t_+ of the mass velocity or dynamic pressure would be greater than the positive duration t_+ of the overpressure. The increase may be taken approximately as $2p$ %, where p is the peak overpressure ($\mathrm{lbf/in^2}$). The U.S.A.E.C. book has a diagram which gives parametric curves for the duration of the overpressure and the dynamic pressure. In the region of interest to us, the diagram gives the increase in positive duration at the $10\,\mathrm{lbf/in^2}$ level as about 40 %, and at the $6\,\mathrm{lbf/in^2}$ level as 30 %.

Most of the observations described earlier and used to determine the nuclear explosive yields are unaffected by these considerations about the dynamic pressure; but observations which involved the drag of the blast wind during a substantial part of the positive phase need a correction to the dynamic pressure; and we have done this where necessary by increasing the time scale (including cases with peaking) by 20 % for pulses with peak overpressures of $10\,\mathrm{lbf/in^2}$, and other cases in linear proportion to the starting pressure. We have not made any correction for (43): if we did, the estimates of the nuclear explosive yield made from observations involving the dynamic impulse would be too large by two or three tenths of a kiloton but such a small correction is negligible in our terms.

Dynamics of elastic–plastic cantilever

Consider a uniform bar of length l and mass σ per unit length, under a dynamic transverse force w per unit length. Neglecting rotatory inertia, the equation for the transverse displacement y at distance x from one end is

$$a^2\frac{\partial^4 y}{\partial x^4}+\frac{\partial^2 y}{\partial t^2}=\frac{w}{\sigma}, \tag{45}$$

where $a^2 = EI/\sigma$, E being the Young modulus and I the second moment of the cross-section. In our application w is provided by the time-dependent blast wind, and we measure x from the base of the vertical bar.

This equation has to be considered in two distinct régimes. In the early stages of the dynamic response of the bar it is wholly elastic until the maximum bending moment (which occurs at the base) reaches the limit M_0 at which plastic yielding begins. Then the bar continues to deflect while the base bending moment remains (for a time at least) constant at M_0, the remainder of the bar responding elastically. At later times the limiting bending moment at the base may

change sign, and in fact a succession of reversals of sign may occur. Thus we have to consider the initial elastic and the subsequent plastic régimes in solving the equation of motion.

The initial conditions are, of course, that the displacement and velocity vanish at all points of the bar at zero time.

The boundary conditions at the free end, $x = l$, are that the shear force and bending moment both vanish there; hence

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^3 y}{\partial x^3} = 0 \quad \text{for} \quad x = l \quad \text{and} \quad t \geqslant 0. \tag{46}$$

At the base of the bar the displacement vanishes; hence

$$y = 0 \quad \text{for} \quad x = 0 \quad \text{and} \quad t \geqslant 0. \tag{47}$$

The remaining condition at the base depends on the régime being considered. In the elastic régime (which ends at, say, time t_1) the slope of the bar is zero; thus

$$\partial y/\partial x = 0 \quad \text{for} \quad x = 0 \quad \text{and} \quad 0 \leqslant t \leqslant t_1. \tag{48}$$

In the plastic régime, while the bending moment at the base remains constant (in magnitude and sense), the remaining boundary condition at the base is

$$\frac{\partial^2 y}{\partial x^2} = \frac{M_0}{EI}, \tag{49}$$

where M_0 is the bending moment corresponding to plastic yielding.

In the elastic régime the solution is

$$y = \frac{2}{\sigma} \sum_{v=1}^{\infty} X_v(x)\, Q_v(t). \tag{50}$$

$$X_v = \{\cos\theta_v + \cosh\theta_v\}\{\sin(x\theta_v/l) - \sinh(x\theta_v/l)\} - \{\sin\theta_v + \sinh\theta_v\}\{\cos(x\theta_v/l) - \cosh(x\theta_v/l)\},$$

$$Q_v = \frac{(\cos\theta_v + \cosh\theta_v)\int_0^t w(\tau)\sin\{p_v(t-\tau)\}\,\mathrm{d}\tau}{p_v\theta_v(\cos\theta_v\sinh\theta_v - \sin\theta_v\cosh\theta_v)^2}.$$

$$p_v = a(\theta_v/l)^2, \quad \text{and the } \theta_v \text{ are the roots of} \quad 1 + \cos\theta\cosh\theta = 0;$$

$$\theta_1 = 1.875, \quad \theta_2 = 4.694, \quad \theta_3 = 7.855, \quad \theta_4 = 10.995,$$

$$\theta_5 = 14.137, \quad \theta_6 = 17.279, \quad \theta_v \approx (2v-1)\,\pi/2 \quad \text{for} \quad v > 6.$$

In the plastic régime we are interested in $\partial y/\partial x$ at $x = 0$, which we may denote by $\psi(T)$, where T denotes time measured from the onset of plasticity, i.e. $T = t - t_1$. Also at $T = 0$, let $\eta(x)$ and $\zeta(x)$ be the displacement and velocity of the bar at x, as found from the elastic solution. Then, while the bending moment at the base remains constant in magnitude and sense, we find

$$\psi = -\frac{3M_0T^2}{2\sigma l^3} + \frac{3}{2\sigma l}\int_0^T w(t_1+\tau)\,(T-\tau)\,\mathrm{d}\tau + \frac{3}{l^3}\int_0^l (\eta + \zeta T)\,x\,\mathrm{d}x$$
$$+ \frac{2M_0 l}{EI}\sum_{v=1}^{\infty}\frac{(1+\cos\beta_v\cosh\beta_v)\cos\alpha_v T}{\beta_v^2\sin\beta_v\cosh\beta_v} + \frac{2}{\sigma l}\sum_{v=1}^{\infty}\frac{1}{\alpha_v}\int_0^T w(t_1+\tau)\sin\{\alpha_v(T-\tau)\}\,\mathrm{d}\tau$$
$$-\frac{1}{a}\sum_{v=1}^{\infty}\frac{\int_0^l F_v(\zeta\sin\alpha_v T + \eta\alpha_v\cos\alpha_v T)\,\mathrm{d}x}{\beta_v\sin\beta_v\cosh\beta_v},$$

$$\left.\begin{aligned}F_v &= (\sin\phi_v + \sinh\phi_v)(\cos\beta_v + \cosh\beta_v) - (\cos\phi_v + \cosh\phi_v)(\sin\beta_v + \sinh\beta_v),\\ \phi_v &= (1 - x/l)\,\beta_v, \quad \alpha_v = a(\beta_v/l)^2.\end{aligned}\right\} \tag{51}$$

The β_v are the roots of $\tanh\beta = \tan\beta$.

$$\beta_1 = 3.927, \quad \beta_2 = 7.069, \quad \beta_3 = 10.210, \quad \beta_4 = 13.352, \quad \beta_5 = 16.493, \quad \beta_v \approx (4v+1)\,\pi/4.$$

Something of the physical significance of the six terms in the expression for ψ may be seen by considering them in groups. Take the simple case where the bar is rigid above the plastic hinge: E, a, and α_v become infinite, causing the fourth, fifth, and sixth terms to vanish. For the rigid bar, of course, $\eta = \psi_0 x$ and $\zeta = \omega_0 x$, where ψ_0 is the slope of the bar and ω_0 its angular velocity at $T = 0$. Hence, the third term becomes $\psi_0 + \omega_0 T$; and, since the moment of inertia of the rigid bar about the hinge is $\sigma l^3/3 = J$ say, the first three terms are simply the solution of the usual equation,

$$J\,\mathrm{d}^2\psi/\mathrm{d}T^2 = -M_0 + \tfrac{1}{2}l^2 w(t_1 + T),$$

describing the rotation of a rigid bar under constant retarding torque of magnitude M_0, and load w per unit length, subject to initial conditions that $\psi = \psi_0$ and $\mathrm{d}\psi/\mathrm{d}T = \omega_0$ at $T = 0$.

These literal solutions were both used in the exploratory work prior to machine computation, and the elastic solution was also used in the initial steps of each of the subsequent machine computations which proceeded directly from the differential equation of motion. Because examination of the expression for ψ made it seem likely that the base slope might pass through a succession of shallow turning values before reaching an absolute maximum, the computations for the plastic phase were carried through with a base bending moment of constant magnitude, and sense chosen to oppose the angular velocity of the base.

The numerical solution of the differential equation of motion was programmed by Dr N. E. Hoskin and followed the method described by Conte (1957) for obtaining a stable implicit finite difference approximation to a fourth-order parabolic equation. Putting $\xi = x/a^{\frac{1}{2}}$, the finite difference equation may be written symbolically as

$$\Delta^4 y + r^{-2}\Delta^2 y = (w/\sigma)\,\Delta\xi^4,$$

where $r = \Delta t/\Delta\xi^2$. With the notation $y_{jk} = y(j\Delta\xi, k\Delta t)$ the finite difference representation of the left-hand side was taken in the form

$$\begin{aligned}&y_{j+2,k} - (y_{j+1,k+1} + 2y_{j+1,k} + y_{j+1,k-1}) + 2(y_{j,k+1} + y_{j,k} + y_{j,k-1})\\ &\quad - (y_{j-1,k+1} + 2y_{j-1,k} + y_{j-1,k-1}) + y_{j-2,k} + r^{-2}(y_{j,k+1} - 2y_{j,k} + y_{j,k-1}).\end{aligned}$$

The normal mode solution was used to start the computation. At any subsequent stage there are three unknown quantities specified on the $(k+1)$-line, the five quantities on the k-line and the three on the $(k-1)$-line being known. Most of the computations were made with a time step of 10^{-4} s, the length of the bar being divided into 180 equal space steps.

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